MODELLING OF CONCRETE USING THE BOUNDARY ELEMENT METHOD AND HOMOGENISATION TECHNIQUE

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ABSTRACT

A 2D boundary element method (BEM) is used to simulate the mechanical behaviour of concrete within a multiscale framework considering the concept of representative volume element (RVE). The problem consists of solving the equilibrium problem in terms of displacement fluctuations at meso-scale of the material. For that, numerical analyses are performed using a consistent tangent operator for BEM formulation. A step-by-step analysis is detailed described for the parametric identification of the variables involved in a uniaxial compression test experimentally tested. The Mohr–Coulomb model is adopted to simulate the mortar matrix while elastic behaviour is considered for aggregates. Besides, the microcracking process at interfacial transition zone is modelled by defining additional cohesive-contact finite elements on interfaces between mortar and aggregates. The proposed numerical model shows its capability to perform this kind of analysis comparing with experimental test.
Keywords: multiscale modelling, homogenisation, RVE, boundary elements, concrete.

1 INTRODUCTION

The understanding of the mechanical properties of materials is crucial to obtain the best performance of them. If numerical analyses are associate with experimental ones, a reduced financial cost can be achieved to develop materials to particular uses. The present work focuses on concrete, the most used material in construction worldwide. In order to analyse the mechanical properties of concrete, the compressive strength is one of the most important parameters and deserves special attention. In fact, that parameter is widely used in specifying concrete under technical standards. Several studies have been developed considering concrete as a composite material formed by materials with different mechanical properties and arrangements leading to complex dissipative phenomena. Therefore, studies that carrying out the aspects of the mechanical behaviour of concrete are welcome.

In this context, the modelling of the mechanical behaviour of concrete using the representative volume element (RVE) concept and homogenisation technique has shown its efficiency and accuracy [1]–[3]. This approach provides macro constitutive models using simple constitutive models on the microstructural level. Besides, to solve the equilibrium problem of this kind of formulation, several numerical methods have been used, mainly finite element method (FEM) [4]–[8]. However, few works deal with this problem using boundary element methods (BEM) [6], [9]–[11]. In the present work, a BEM formulation is used to simulate the mechanical behaviour of concrete considering interfacial transition zone (ITZ) modelled by cohesive fracture elements and adopting a standard Mohr–Coulomb model and elastic behaviour for mortar matrix and aggregates, respectively. Besides, porous are inserted in the mortar matrix. In other works (Fernandes et al. [10], [11]), BEM has proved its efficiency when compared to others numerical models leading to reliable results but with smaller computational effort.

Finally, our goal is to show the potentialities of BEM when applied in practical situations as presented as in this work dealing with parametric identification of a concrete experimentally tested.

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2 MULTISCALE BOUNDARY ELEMENT APPROACH

A BEM formulation detailed discussed in Fernandes et al. [10] and Fernandes and Souza Neto [12] has been used on the development of a multiscale approach based on the RVE concept. In this multiscale boundary element approach (MBEA), the 2D-dimensional plate problem is defined to model the RVE on the microstructure level. Also, the BEM equations are used to perform the analysis related to macrocontinuum. The communication between scales is made by computational homogenisation techniques detailed in Santos et al. [7], Santos and Pituba [8] and Borges and Pituba [13].

In case of small strain problems considering plasticity phenomenon, the total strain $\dot{\varepsilon}_{ij}$ is split into its elastic and plastic parts,$\dot{\varepsilon}_{ij}^{e}$ and $\dot{\varepsilon}_{ij}^{p}$, respectively. On the other hand, when phase debonding fracture processes are considered, the total strain $\dot{\varepsilon}_{ij}$ is split into the continuum $\dot{\bar{\varepsilon}}_{ij}$ and the fracture strains $\dot{\varepsilon}_{ij}^{cf}$. Therefore, the strain field in the RVE is defined as [14]:

$$\dot{\bar{\varepsilon}}_{ij} + \dot{\varepsilon}_{ij}^{cf} = \dot{\varepsilon}_{ij}^{e} + \dot{\varepsilon}_{ij}^{p}. \tag{1}$$

The Betti's theorem is used to define the integral representation of displacement. For a particular region of the plate composed of different materials, the Betti´s theorem can be written as follows (see more details in Fernandes et al. [10], [11] and Silva et al. [14]):

$$\int_{\Omega} \varepsilon_{kij}^{*}\left(\dot{N}_{ij} + \dot{N}_{ij}^{0}\right) d\Omega = \sum_{m=1}^{N_S}\left[\frac{\overline{\overline{E}}_m}{\overline{\overline{E}}}\frac{\nu_m}{\nu}\int_{\Omega_m}\dot{\bar{\varepsilon}}_{ij}N_{kij}^{*}d\Omega + \overline{E}_m\left(1-\frac{\nu_m}{\nu}\right)\int_{\Omega_m}\dot{\bar{\varepsilon}}_{ij}\varepsilon_{kij}^{*}d\Omega_m\right]$$

$$k, i, j = 1, 2, \tag{2}$$

in which terms with the superscript * relate to the fundamental problem, $\overline{\overline{E}}_m = \frac{\bar{E}_m}{(1-\nu_{m^2})}$, $\bar{E}_m = E_m t$, E is the Young's modulus; ν, the Poisson's ratio; t, the plate thickness; δ_{ij}, the Kronecker's delta; N_s is the number of subregions; $\dot{N}_{ij}$ is the membrane force obtained from the stress tensor rate $\dot{\sigma}_{ij}$, obtained by the plastic criterion; inelastic forces $\dot{N}_{ij}^{0}$ are related to plasticity and phase debonding phenomena and k, the fundamental load direction; ν, $\overline{E}$, and all fundamental values refer to the subregion in which the collocation point is placed.

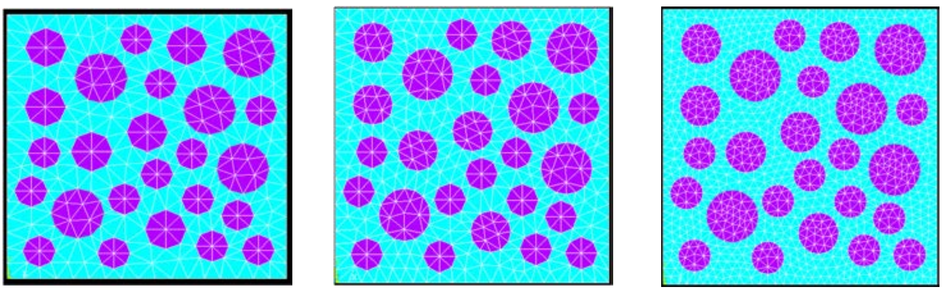
On the other hand, the Hill–Mandel Principle is used to define the RVE equilibrium problem. This Principle guarantees the energy consistency between scales. After several mathematical manipulations and the RVE discretisation into N_{cel} cells, the principle leads to the equilibrium eqn (3) considering the displacement field obtained from the macroscopic strain $\varepsilon(x)$ imposed to the RVE boundary and the displacement fluctuation field $\tilde{u}_{\mu}$.

$$R_F = \sum_{e=1}^{N_{cel}} B_e^T \Delta\sigma_{\mu}^{e(i)} t\, A_e = \sum_{e=1}^{N_{cel}} [B]_e^T t\left[D_{\mu}^{e}\right](\{\Delta\varepsilon\} + [B]\{\Delta\tilde{u}\})_e A_e = 0, \tag{3}$$

in which $\left[D_{\mu}^{e}\right]$ is the constitutive tensor related to the cell e, B_e, the cell strain-displacement matrix, A_e, the cell area; and $N_{\mu}^{e} = t\sigma_{\mu}^{e}$, the cell membrane force increment vector, considered constant over the cell.

3 NUMERICAL APPLICATION: PARAMETRIC IDENTIFICATION OF CONCRETE USING MBEA

In this section is presented a parametric identification of a concrete experimentally tested by Delalibera [15] using the MBEA described in Section 2. Accordingly with experiment tests carried out by Delalibera [15], 40.61% of aggregate volume fraction has been adopted for the RVE distributed as five aggregates with 19 mm of diameter, six aggregates with 15 mm and 14 aggregates with 12 mm. The RVE adopted is shown in Fig. 1. Also, it is adopted an elastic behaviour for the aggregates, Mohr–Coulomb criterion to model the mortar matrix, both discretised by triangular element cells while a contact and cohesive fracture model is used in the ITZ modelled by interface elements (see Table 1). In all analyses, a macrostrain tensor is imposed to the RVE ($\varepsilon_x = -0.003$; $\varepsilon_y = 0.0006$ and $\varepsilon_{xy} = 0$), which is divided into 20 increments adopting periodic fluctuations to the RVE boundary as discussed in Santos et al. [7], Santos and Pituba [8], Fernandes et al. [10] and Borges and Pituba [13]. The goal of this section is to evidence the potentialities of the MBEA in representing the concrete mechanical behaviour submitted to uniaxial compression stress state capturing the major features of this material when compared to the experimental curve presented in Delalibera [15].



(a) (b) (c)

Figure 1: RVE discretisation. (a) RVEa with 634 cells; (b) RVEb with 564 cells; and (c) RVEc with 1276 cells.

Table 1: Properties of the mortar matrix, aggregates and ITZ.

Phase	Properties				
	ν	**E (GPa)**	**Adopted constitutive model**	**Hardening points** $(\bar{\varepsilon}_p, \sigma_y(MPa))$	**Friction angle (ϕ)**
Mortar matrix	0.2	23	Mohr–Coulomb	(0;11) (0.22;40)	4°
Aggregates	0.3	40	Elastic	–	–
	β_c	σ_c **(MPa)**	δ_c **(mm)**	λ_p	
ITZ	0.707	1.00	0.0568	200,000.00	

First of all, three different discretisation have been used to model the problem in order to achieve the objectivity of the numerical response. Table 2 contains information about the discretisation. Fig. 2 shows the homogenised stress in the *x* direction (σ_x^{hom}) versus imposed macrostrain in the *x* direction curves obtained by different meshes. As it can see, the meshes show the same results. In particular, the poorest mesh is then considered for further analyses. Note that the using of cohesive and contact finite elements have been adopted only on the interfaces between the matrix and bigger aggregates overcoming instability problems inherent to this kind of element.

Table 2: Information on the discretisation of the RVEs.

RVE	Number of nodes, elements and cells					
	Nodes	Cells	Boundary elements	Interface nodes	Interface elements	Cohesive and contact elements
RVEa	412	634	68	280	220	60
RVEb	564	882	84	344	264	80
RVEc	1276	2214	136	520	420	100

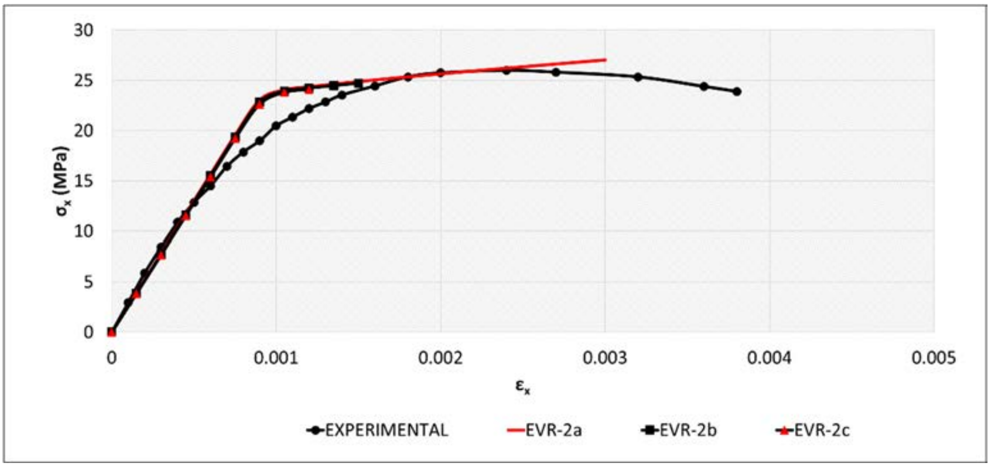
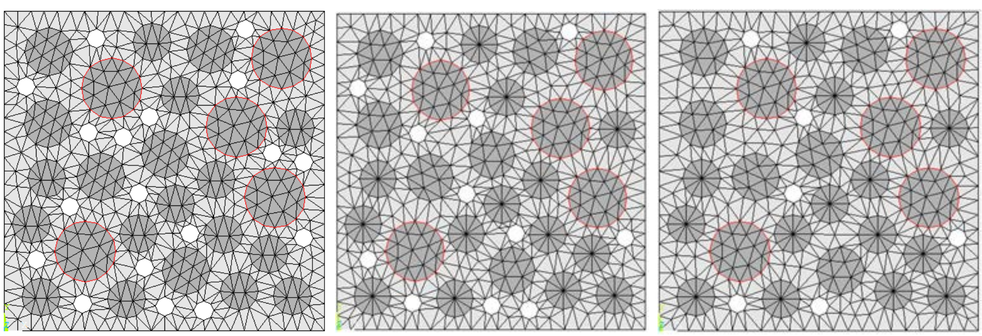


Figure 2: Homogenised stress and imposed macrostrain strain curves for RVEa, RVEb and RVEc.



(a) (b) (c)

Figure 3: RVEs with different porosity fractions. (a) 5%; (b) 3.34%; and (c) 2%. Red lines around the bigger aggregates indicate the use of contact and cohesive fracture elements.

To improve the numerical response around 15 MPa, when the microcracking process starts to play an important role on the dissipative phenomena, the porosity of the mortar matrix must be considered. Besides, this kind of porosity is observed experimentally. Therefore, a study is carried out considering the insertion of porous following the proportions presented in Fig. 3. Fig. 4 shows the results of the RVE-0% (former RVE described in Fig. 2), RVE-2%, RVE-3.34% and RVE-5%. Note that the results are improved in the pre-peak region, but a strength reduction is observed, as expected. As it can see, the RVE-3.34% represents better the experimental response.

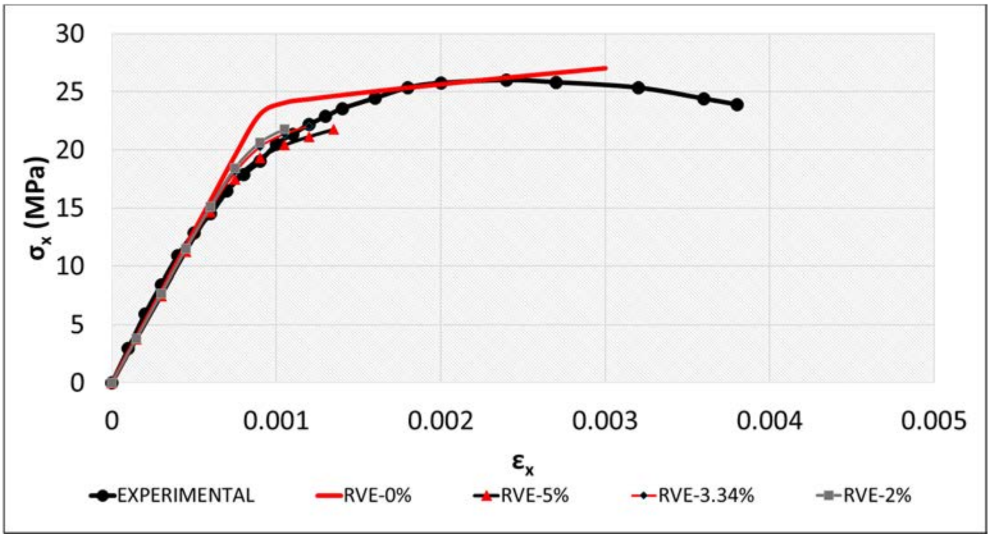


Figure 4: Homogenised stress in the x direction versus imposed macrostrain for RVE-0%, RVE-2%, RVE-3.34% and RVE-2%.

Finally, the RVE-3.34% model leads to a reduced strength of the homogenised material. In order to overcome this issue a change of isotropic hardening curve related to Mohr–Coulomb model for mortar matrix is made (see Table 1). The new parameters of the isotropic hardening curve ($\bar{\varepsilon}_p, \sigma_y(MPa)$) are given by (0;11) and (0.22;70). The new RVE is called RVE-3.34%F. Fig. 5 shows results for the RVE-3.34% and the new one RVE-3.34%F. Note that the homogenised macroscopic model predicts the experimental response quite well until the peak stress zone obtaining the experimentally observed concrete strength. Note that a different RVE modelling could be proposed in which microcracks could be created and propagated inside the mortar matrix to capture the softening behaviour evidenced in pos-peak region.

4 CONCLUSIONS

In the present work, a 2D-dimensaional formulation is presented using the BEM and computational homogenisation technique within a multiscale approach. In this work, the MBEA has been used in numerical analyses just on the mesoscale level considering RVE of concrete previously experimental tested by Delalibera [15]. The MBEA has proved to be a feasible alternative to perform analyses of concrete capturing the ultimate compressive

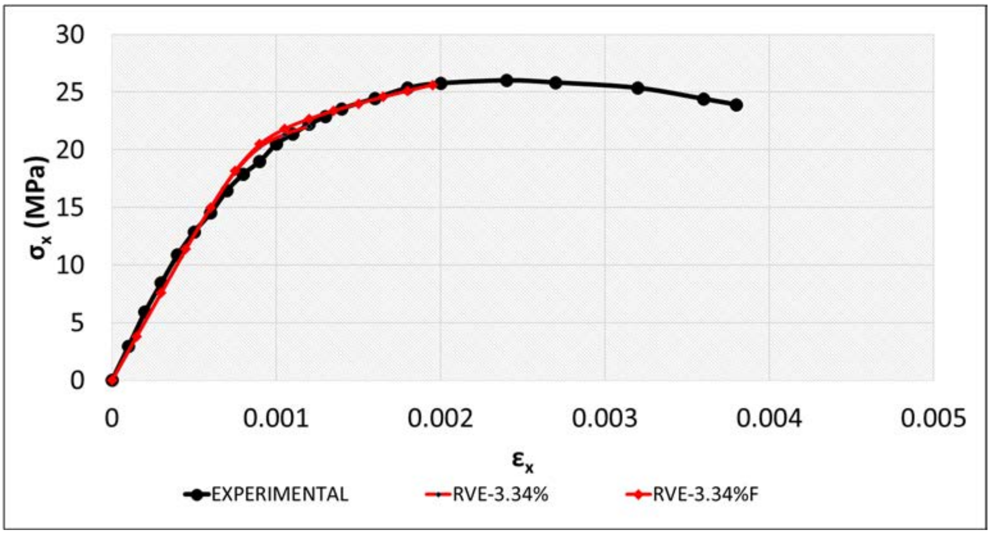


Figure 5: Homogenised stress in the x direction versus imposed macrostrain for RVE-3.34% and RVE-3.34%F.

strength of the material as well as the major features of its mechanical behaviour. Besides, the porosity in the mortar matrix must be considered in the homogenised macroscopic model despite the reduction of the strength evidenced by the model. In fact, this reduction is experimented by the material and the porosity has important role in this phenomenon.

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