FINITE ELEMENT METHOD/BOUNDARY ELEMENT METHOD-BASED MICROSTRUCTURAL TOPOLOGY OPTIMIZATION OF SUBMERGED BI-MATERIAL THIN-WALLED STRUCTURES

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ABSTRACT

Periodic microstructures are often found in structures in the field of vibration acoustics and topology optimization is an effective method for the design of microstructures. Microstructural topology optimization of bi-material for minimizing the responses of the exterior acoustic-structure interaction system is investigated. The structural finite element method is combined with the acoustic boundary element method to analyse the response of the acoustic-structure interaction system, and the bi-directional coupling of the acoustic-structure system is considered. The equivalent macroscopic elastic matrix of microstructures is calculated by homogenization method. Topology optimization model is schemed based on the piecewise constant level set method. Considering the high efficiency of adjoint variable method in multi-variable and high complexity optimization problems, this research adopts the adjoint variable method to analyse the sensitivity of the objective function of the coupled system. Numerical results show that the response of the coupled systems can be reduced significantly, indicating the effectiveness of the optimization algorithm.

Keywords: microstructural, topology optimization, boundary element method, finite element method, PCLS.

1 INTRODUCTION

Reducing vibration acoustic radiation of thin-walled structures is an important problem in engineering. Some structures applied in the field of vibration acoustics often have micro-structures with periodic arrangement. For such structures, the quality of the micro-structure configuration plays a decisive role in the vibration acoustic performance of the structure. Topology optimization can make full use of the potential of structure and material, and it is an effective method for the design of microstructure.

In recent years, there are a lot of researches on macroscopic topology optimization based on vibrational or acoustic criteria, but relatively few researches on material microstructure topology optimization. Yang and Du [1], [2] established a topological optimization model of material microstructure based on minimizing the radiated acoustic power of macroscopic structure surfaces. The results showed that the topological optimization of material microstructure could reduce the radiated acoustic power. They used the high frequency approximation method to get the sound pressure on the surface of the structure, avoiding solving the coupling equation. In addition, in the case of air, the effect of the sound field on the structure can be ignored, so they used the weak coupling condition. Considering the strong coupling between structure and sound field, Chen et al. [3], [4] established the robust micro-structure topology optimization model of the internal sound field coupling system by using finite element method (FEM). However, for the external sound field, it is difficult to apply the boundary conditions and the discretization of the field is too large by using FEM. For the exterior acoustic-structure interaction system, the FEM/boundary element method (BEM) coupling algorithm is a better solution, Zhao et al. [5] proposed a bi-material topology

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optimization approach based this method. Following this line, a microstructural topology optimization method based on FEM/BEM is proposed in this research.

2 STRUCTURAL-ACOUSTIC ANALYSIS

The macroscopic equivalent elastic constant matrix D^H of microstructure can be obtained by the homogenization method [6], [7].

$$D^H = \frac{1}{|Y|}\sum_{e=1}^{n_e}\int_{Y_e} D^{MI}(I - bu_e)dY, \tag{1}$$

where $|Y|$ denotes the volume of a microcell, n_e denotes the total number of cell units, D^{MI} denotes the element elastic matrix of a cell, I denotes the identity matrix, b denotes the strain matrix of a cell, u_e denotes the element displacement matrix of a cell.

The equivalent material elastic matrix D^H can be incorporated into the FEM part of the FEM/BEM coupling method. And the derivative part can be calculated by substituting the following formula into the FEM part.

$$\frac{\delta D^H}{\delta x} = \frac{1}{|Y|}(\int_{Y_e} (I - bu_e)^T \frac{\delta D^H}{\delta x}(I - bu_e)dY). \tag{2}$$

Considering the impact of sound field on the structure, the finite element discrete equation of the structure is

$$(\boldsymbol{K} - \omega^2\boldsymbol{M})\boldsymbol{u} = \boldsymbol{A}\boldsymbol{u} = \boldsymbol{f} + \boldsymbol{C_{sf}}\boldsymbol{p}, \tag{3}$$

where $\boldsymbol{K}$ and $\boldsymbol{M}$ denote respectively the stiffness and the mass matrices of the structure, $\boldsymbol{u}$ denotes the nodal displacement vector, $\boldsymbol{f}$ denotes the nodal force vector and $\boldsymbol{C_{sf}} = \int \boldsymbol{N}_s^T\boldsymbol{n}\boldsymbol{N}_f\, d\Gamma$ denotes the coupling matrix, $\boldsymbol{N_f}$ and $\boldsymbol{N_s}$ denote the shape functions for the fluid and structural domains, and $\boldsymbol{n}$ is the unit normal vector on the boundary Γ. $\boldsymbol{p}$ denotes the sound pressure vector. The acoustic domain which is based on the BEM formulation can be written as

$$\boldsymbol{H}\boldsymbol{p} = \boldsymbol{G}\boldsymbol{q} + \boldsymbol{p_{in}}, \tag{4}$$

where $\boldsymbol{H}$ and $\boldsymbol{G}$ are respectively the BEM coefficient matrices, $\boldsymbol{q}$ and $\boldsymbol{p_{in}}$ denote the flux and incident wave vectors. The governing equations, eqns (3) and (4), are linked via the continuity condition:

$$\boldsymbol{v} = -i\omega\boldsymbol{S}^{-1}\boldsymbol{C_{fs}}\boldsymbol{u}, \tag{5}$$

where $\boldsymbol{v}$ is the normal vibration velocity of the acoustic medium at the boundary Γ, $\boldsymbol{S} = \int \boldsymbol{N}_f^T\boldsymbol{N}_f\, d\Gamma$, and $\boldsymbol{C_{fs}} = \boldsymbol{C}_{sf}^{\mathrm{T}}$. According to $\boldsymbol{q} = i\omega\rho\boldsymbol{v}$, eqns (3) and (4) can be written as the following formulation

$$(\boldsymbol{H} - \boldsymbol{G}\boldsymbol{W}\boldsymbol{C_{sf}})\boldsymbol{p} = \boldsymbol{G}\boldsymbol{W}\boldsymbol{f_s} + \boldsymbol{p_{in}}, \tag{6}$$

where $\boldsymbol{W} = \omega^2\rho\boldsymbol{S}^{-1}\boldsymbol{C_{fs}}\boldsymbol{A}^{-1}$. The iterative solver GMRES and PARDISO package in Intel Math Kernel Library (MKL) are used to solve the above equation. After solving the equation, the acoustic and structural response is obtained. After discretization, the sound power P can be expressed as follows

$$P = \frac{1}{2}\boldsymbol{\Re}(\boldsymbol{p}^{\boldsymbol{T}}\boldsymbol{S}\boldsymbol{v}^*), \tag{7}$$

where $\boldsymbol{\Re}$ denotes the real part of complex value, and $()^*$ denotes the conjugate transpose.

3 MINIMIZATION OF VIBRO-ACOUSTICS RESPONSE USING TOPOLOGY OPTIMIZATION

The vibro-acoustics response of coupling system can be described by structural displacement u and sound field pressure p or a function of u and p. The sound power level is taken as the objective function. The problem can be formulated as follows:

$$\min \Pi = 10\log\frac{P}{P_0}$$
$$\text{s.t.} \sum_{e=1}^{N} \rho_e V_e - f_v \sum_{e=1}^{N} V_e \leq 0, \tag{8}$$
$$\rho_e \in \{0, 1\}$$

where P_0 is the reference sound power, ρ_e is the design variable, 0 and 1 represent the regions of material 1 and material 2, f_v denotes the volume fraction constraint.

Details of calculating sensitivity using adjoint variable method can be referred to [5]. The piecewise constant level set method is employed to solve the optimization problem [8]. When the relative change of the objective function value is less than a specified value, the iteration terminates.

4 NUMERICAL EXAMPLES

A numerical example is presented to illustrate the validity of the optimization method in this section. Considering a submersed hexahedral thin-walled box as shown in Fig. 1. The dimensions of the cube are 1 m × 1 m × 1 m with the thickness 0.01m. The top surface of the cube discretized by Kirchhoff plate elements and the other surfaces are rigid. The steel is chosen as the Material 1 (E_1=210 GPa, ρ_1 = 7800kg/m^3, ν_2 = 0.3), and Material 2 has the properties E_2 = 0.1E_1, ρ_2 = 0.1ρ_1, and ν_2 = ν_1. The harmonic excitation is set to be F = 1,000 N, f_p = 60 Hz, and the volume ratio of material 1 to material 2 is 1:1.

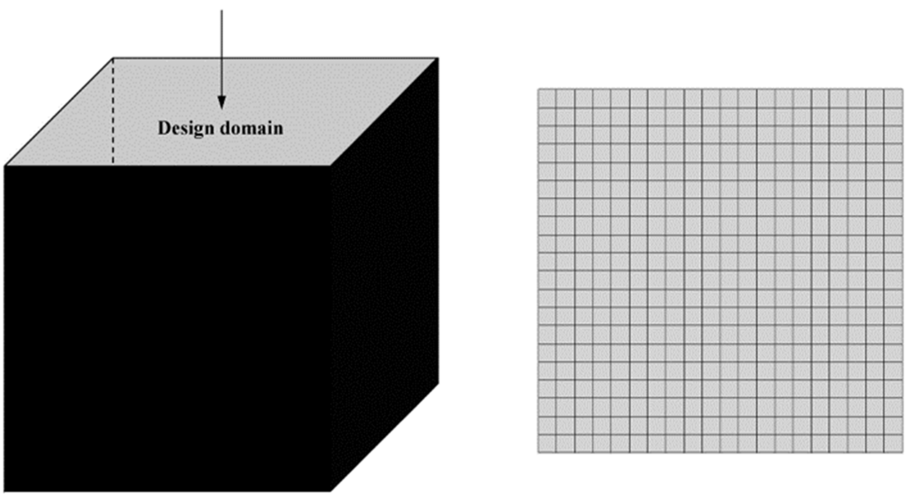

Figure 1: The hexahedral structure, model and mesh.

Fig. 2 shows the optimized microstructural unit cells, and Fig. 3 shows the sound pressure level (SPL) of the structure surface. As can be seen from Fig. 3 that the SPL on the surface of the structure is reduced after optimization, and even lower than that when all material 1 with higher stiffness are used, which indicates that the optimization is effective.

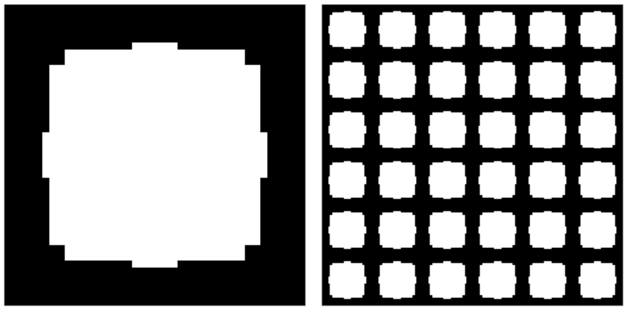

Figure 2: The optimized microstructural unit cell and the corresponding 6 × 6 arrays.

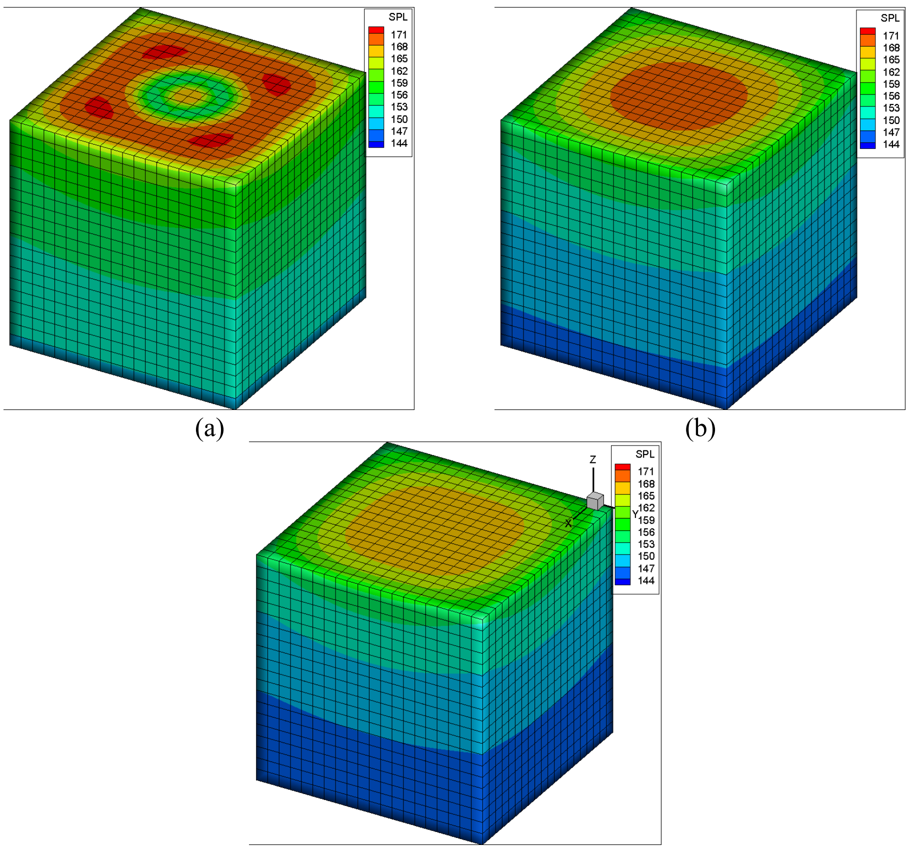

Figure 3: The sound pressure level of the structural surface. (a) Before optimization; (b) Full of material 1; and (c) After optimization.

5 CONCLUSIONS

In this paper, the microstructural topology optimization technique for bi-material structure has been presented to find the optimum microstructural distribution which reduces the responses of the exterior acoustic-structure interaction systems. The effect of sound field on the structure is considered in optimization. Numerical examples show that this method can reduce the response of acoustic-structure interaction systems.

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