Dynamic simulation of heat conduction using a BEM model in the frequency domain: an experimental validation

I. Simões[1], N. Simões[1], A. Tadeu[1] & M. Reis[2]
[1]CICC, Department of Civil Engineering, Faculty of Sciences and Technology, University of Coimbra, Portugal
[2]Department of Fortification and Construction Engineering, Military Institute of Engineering, Rio de Janeiro, Brazil

Abstract

The heat transfer by conduction across a system containing heterogeneities, when subjected to unsteady conditions, is simulated using a boundary element method (BEM), formulated in the frequency domain. The proposed formulation is validated experimentally.

In the laboratory tests a steel inclusion was embedded in a confined host medium and unsteady temperatures were applied to its boundary. The thermal properties of the host media tested (molded expanded polystyrene) has been previously defined experimentally. The results showed that the BEM solutions agree well with the experimental results.
Keywords: conduction, transient heat transfer, BEM; experimental validation, frequency domain, Green's functions.

1 Introduction

As the analytical solutions for simulating heat diffusion are only known for very simple geometric and material conditions, such as homogeneous full spaces, half-spaces and circular cylindrical inclusions, various numerical techniques have been proposed for more complex problems. Unsteady state conditions can be split into three formulation groups: time domain; transformed domain, by applying the Laplace transform, and frequency domain, by applying a Fourier Transform.

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In the frequency domain technique, a Fourier transform applied in the time domain deals with the time variable of the diffusion equation to establish a frequency domain technique; time solutions are obtained by using inverse Fourier transforms into time-space. The time-aliasing phenomenon is avoided by using complex frequencies to attenuate the response at the end of the time frame. This effect is later taken into account by re-scaling the response in the time domain [1].

The BEM is usually one of the tools for modelling heat transfer in heterogeneous media, requiring only the discretization of the inclusions' boundaries. Hohage and Sayas have proposed a numerical solution of the heat diffusion problem using the BEM applying the Laplace transform [2]. Other authors study the transient heat conduction with BEM, for example, Ma *et al.* [3] and Wang *et al.* [4]. Indeed, Tadeu *et al.* have recently proposed an algorithm that couples the BEM and the method of fundamental solutions (MFS) to study transient heat diffusion by conduction across heterogeneous media containing thin inclusions [5].

In this work, it is described the experimental validation of a BEM formulation for studying unsteady heat transfer by conduction, across a system containing heterogeneities. After a brief description of the BEM formulation, the model is validated against experimental results. The laboratory test setup is briefly described.

2 Boundary element method formulation

For simplicity, consider an unbounded medium (Medium 1) with an embedded inclusion (Medium 2), bounded by a surface S, as illustrated in Figure 1). Null fluxes and prescribed temperatures $T_0(t)$ are imposed along the boundary sections S_1 and S_2, respectively, while continuity of temperatures and heat fluxes are assumed along the remaining part of the boundary, S_3, ($\{S_1, S_2, S_3\} \in S$).

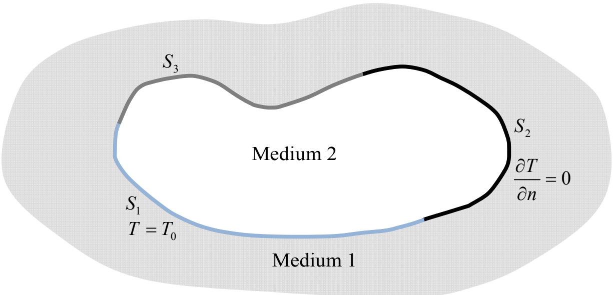


Figure 1: Geometry of the problem.

The transient heat transfer by conduction is governed by the equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) T(t,x,y) = \frac{1}{K} \frac{\partial T(t,x,y)}{\partial t} \tag{1}$$

in which t is time, $T(t,x,y)$ is temperature, $K = \lambda/(\rho c)$ is the thermal diffusivity, λ is the thermal conductivity, ρ is the density and c is the specific heat.

To solve this equation we move from the time domain to the frequency domain by applying a Fourier transformation in the time domain to eqn (1). Performing the integration by parts, we get

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + {k_\alpha}^2\right)\hat{T}(\omega,x,y) = 0 \tag{2}$$

where $\hat{T}(\omega,x,y) = \int_0^\infty T(t,x,y)e^{-\mathrm{i}\omega t}\,dt$, $k_\alpha = \sqrt{-\mathrm{i}\omega/K}$, $\mathrm{i}=\sqrt{-1}$ and ω is the frequency.

The boundary integral equation can be constructed by applying the reciprocity theorem [6], leading to

a) along the exterior domain of the inclusion (Medium 1):

$$\begin{aligned} c\hat{T}^{(1)}(x_0,y_0,\omega) = {} & \int_{S_3} q^{(1)}(x,y,\eta_n,\omega)G^{(1)}(x,y,x_0,y_0,\omega)\,ds \\ & -\int_{S_3} H^{(1)}(x,y,\eta_n,x_0,y_0,\omega)\hat{T}^{(1)}(x,y,\omega)\,ds \end{aligned} \tag{3}$$

b) along the interior domain of the inclusion (Medium 2):

$$\begin{aligned} c\hat{T}^{(2)}(x_0,y_0,\omega) = {} & \int_{S_2+S_3} q^{(2)}(x,y,\eta_n,\omega)G^{(2)}(x,y,x_0,y_0,\omega)\,ds \\ & -\int_{S_1+S_3} H^{(2)}(x,y,\eta_n,x_0,y_0,\omega)\hat{T}^{(2)}(x,y,\omega)\,ds - \int_{S_2} H^{(2)}(x,y,\eta_n,x_0,y_0,\omega)\hat{T}_0(x,y,\omega)\,ds \end{aligned} \tag{4}$$

In these equations, the superscripts (1) and (2) refer to the exterior and interior domains respectively, η_n is the unit outward normal along the boundary, G and H are respectively the fundamental solutions (*Green*'s functions) for the temperature ($\hat{T}$) and heat flux (q), at (x,y) due to a virtual point heat load at (x_0,y_0). The factor c is a constant defined by the shape of the boundary, taking the value $1/2$ if the shape is smooth and $(x_0,y_0)\in S_1$ or $(x_0,y_0)\in S_3$.

The required Green's functions for temperature and heat flux in Cartesian coordinates are given by

$$G(x,y,x_0,y_0,\omega) = \frac{-\mathrm{i}}{4\lambda}H_0\left(k_\alpha\, r\right) \tag{5}$$

$$H\left(x,y,\eta_n,x_0,y_0,\omega\right) = \frac{\mathrm{i}}{4\lambda}k_\alpha H_1\left(k_\alpha\, r\right)\frac{\partial r}{\partial \eta_n} \tag{6}$$

in which $r=\sqrt{\left(x-x_0\right)^2+\left(y-y_0\right)^2}$ and $H_n(\ )$ are Hankel functions of the second kind and order n.

The final system of equations was assembled so as to ensure the continuity of temperatures and heat fluxes along S_3. This requires the discretization of the interface S. The unknown nodal temperatures and heat fluxes were obtained by solving this system of equations, allowing the heat field in the domain to be defined.

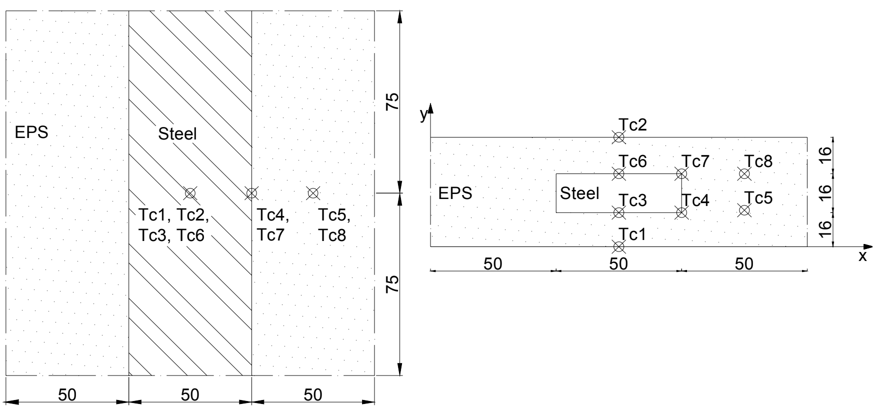
The integrations in eqns (3) and (4) were evaluated using a Gaussian quadrature scheme when the element to be integrated is not the loaded element, while for the loaded element, the existing singular integrands in the source terms of the *Green*'s functions were calculated in closed form (see Tadeu *et al.* [7]).

3 Experimental validation

The results provided by the BEM model were compared with those obtained experimentally using two systems comprising a steel inclusion embedded in a confined homogeneous medium, when subjected to unsteady heat flow rates. Thermocouple sensors connected to a data logger system are used to record the temperature change within the test specimen. The measurements were later compared with those computed by the BEM model using as input data the external surface temperature changes and the thermal properties of each material, which were obtained experimentally.

3.1 Experimental tests

The system measured 500x500 mm^2, was 48 mm thick and hosted a steel element measuring 330x50 mm^2, by 16 mm thick (see Figure 2). The host media was molded expanded polystyrene (EPS). This material was characterized beforehand to determine its thermal conductivity (found by the *Guarded Hot-Plate Method*) (ISO 8302:1991 [8], following the test procedure defined in



a) b)

Figure 2: Measurement zone of specimens tested and position of thermocouples: a) plane view; b) cross section.

EN 12667:2001 [9]), mass density (EN 1602:1996 [10]) and specific heat (following the ratio method), with at least three measurements. Table 1 lists those properties, and they were used in the numerical simulations.

Table 1: Materials' thermal properties.

Material	Conductivity, k ($W.m^{-1}.^{o}C^{-1}$)	Mass density, ρ ($kg.m^{-3}$)	Specific heat, c ($J.kg^{-1}.^{o}C^{-1}$)
Molded Expanded Polystyrene (EPS)	0.040	14.3	1430.0
Steel	53.0	7850.0	450.0

The experimental system was subjected to an unsteady heat flow rate imposed by a single-specimen Lambda-meter EP-500 apparatus (see Figure 3). Before running any test, the specimen was conditioned in a climatic chamber, Fotoclima 300EC10 from Aralab, in a controlled environment with a set-point temperature of (23±2) ºC and (50±5) % of relative humidity, until constant mass was reached. These were also the conditions in the room environment where tests were run. In the unsteady measurement test the single-specimen Lambda-meter EP-500 was first programmed to reach a temperature of 23ºC in the middle of the test specimen, establishing a 15ºC temperature difference between the heating and the cooling units. The equipment ensures null heat flows across lateral sides in contact with the room environment. The energy input was supplied until a permanent heat flow rate was reached. Then, the heating and cooling units were switched off and the system was allowed to regain energy equilibrium with the room environment.

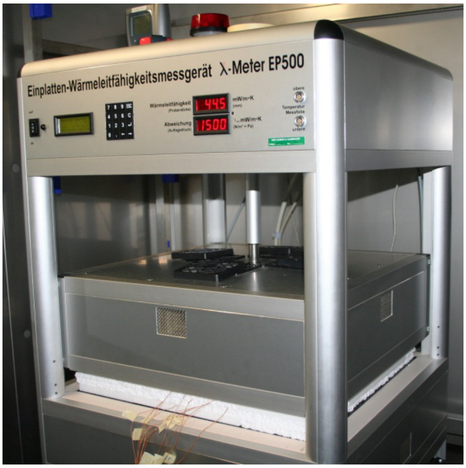


Figure 3: Single-specimen Lambda-meter EP-500 apparatus with a tested system inside.

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The temperature variation at each selected point (see Figure 2) was measured using type T (copper) thermocouples made of 0.2 mm diameter wire, placed on each outer surface of the system, on the top and bottom of the inclusion interface, and in the host material. The data were recorded using a Yokogawa MW 100 data logger, with a time interval of 10 seconds.

3.2 BEM simulation

In the BEM simulation, the temperatures recorded by thermocouples on the top surface – T_{0t} – and the temperatures registered in bottom plate – T_{0b} – were first transformed to frequency domain by applying a direct discrete fast Fourier transform in the time domain. Note that those temperatures were reduced by subtracting the initial test temperatures.

Afterwards the heat field was computed in the frequency domain. The system was computed in the frequency range $0.0\,\text{Hz}$ to $0.028635\,\text{Hz}$, with a frequency increment of 1.3982×10^{-5}Hz, which determined a full analysis window of $19.9\,\text{h}$. The responses were computed at the eight receivers that were in the same tested positions as the thermocouples.

The temperature in the time domain was found by applying a discrete inverse fast Fourier transform in the frequency domain. The aliasing phenomena were dealt with by introducing complex frequencies with a small imaginary part, taking the form $\omega_c = \omega - \mathrm{i}\eta$ (where $\omega = 0.7\Delta\omega$, and $\Delta\omega$ is the frequency increment). This shift was subsequently taken into account in the time domain by means of an exponential window, $e^{\eta t}$, applied to the response.

The final temperatures were obtained by adding the initial test temperatures to these responses.

3.3 Comparison of the results

The experimental measurements were presented and compared with the results computed with the BEM formulation. In the figure below the solid lines are the BEM responses while the experimental measurements are represented by the lines with marked points. Temperature changes at the 8 receivers are plotted in Figure 4 for a time window of 16 hours. Higher and lower temperature lines are related to the top and bottom temperature surfaces that were used as input data for the BEM simulations.

Analyses of these responses show a good agreement between the BEM responses and the experimental results.

All thermocouples and simulation receivers exhibit similar initial temperatures of around (23±2) ºC – temperature in the room. Once the apparatus has started emitting energy, the top and bottom surfaces register a change of temperature so that a difference of 15ºC is established between them. This difference is maintained until a steady state is achieved.

Observing the behavior at a cross-section through the middle of the steel inclusion (see Tc2, Tc6, Tc3 and Tc1), very small temperature differences are

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found between Tc3 and Tc6, since steel allows higher heat flow rates than the host media. Tc4 and Tc7 at the corners of the steel show higher temperature differences but lower than Tc5 and Tc8, which are further away from the steel inclusion.

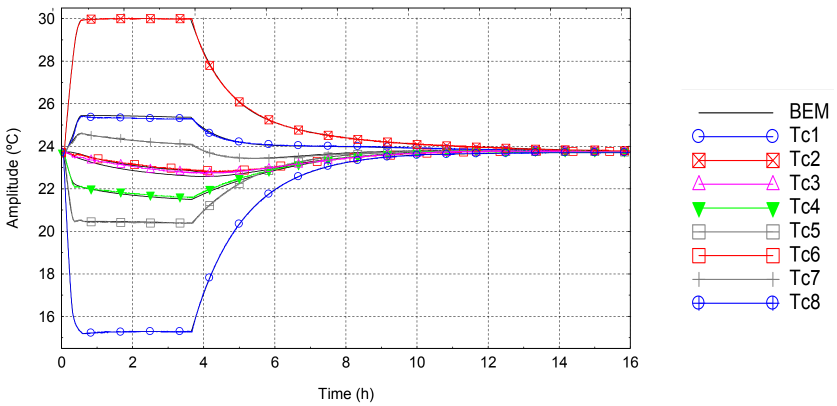

Figure 4: Temperature curves plotted for the experimental measurements and the BEM simulations.

Given the thermal property differences between the materials used in each system, we would underline the good agreement that was found between the experimental measurements and results obtained with the BEM model. Thus, the BEM algorithm based on a frequency domain formulation may be considered adequate to simulate the heat transfer in heterogeneous systems.

4 Conclusions

A BEM model has been implemented and used to compute the two-dimensional transient heat conduction in heterogeneous systems. This model is based on a frequency domain formulation: a Fourier transform applied in the time domain was the technique used to deal with the time variable of the diffusion equation. The heat field was established by imposing the continuity of temperature and heat fluxes at the interfaces of the heterogeneities while verifying the exterior boundary conditions.

The BEM model was verified and validated by comparing its solutions with analytical solutions and experimental measurements. The experimental validation used one parallelepiped heterogeneous system composed of a steel inclusion embedded in molded expanded polystyrene. Temperature variations were imposed at the top and bottom exterior surfaces and null heat fluxes at the lateral surfaces of the systems. We can conclude that when the thermal proprieties of the materials are known the proposed BEM formulations in the frequency domain may be considered consistent for the purpose of studying transient heat conduction in systems with heterogeneities.

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