# Optimum design of cut off walls located in the foundation of diversion dams using boundary element method

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#### Abstract

Foundation design for hydraulic structures founded on permeable soils has been a concern among designers. Estimation of uplift forces and hydraulic gradient at any point, especially at key locations of a permeable media is one of important elements in designing concrete gravity dams on such foundation conditions. Among different numerical methods, the Boundary Element Method (BEM) employed for solving Laplacian equation of flow through porous media has been proved to be a powerful and effective tool for design purpose. The precision and speed of this method to evaluate the hydraulic gradient and potential at any point of foundation enable designers to find a suitable location of cutoff walls and filters.

In this investigation, the impression of various parameters such as length and position of cutoff walls, position of filter, depth of porous foundation and type of soils has been considered and their effects on uplift pressure and hydraulic gradient are represented graphically. In particular, the results obtained from the present work clearly indicate that placing an extra cutoff wall between the two lateral walls has no considerable effect on the uplift force or hydraulic gradient, whereas locating a proper filter in a suitable place on the foundation can remarkably change the flow pattern and affect the design process. Results of this study could be of the great help for engineers to design diversion dams by efficient methods.

Keywords: BEM, diversion dam, uplift pressure, hydraulic gradient, cut off wall, filter.



## 1 Introduction

Installation of hydraulic structures in areas where water penetration under facilities is probable requires great care due to instabilities caused by the leakage of water. Mentioned instabilities occur mainly due to two reasons, the first is uplift pressure development and change in the equilibrium of effective forces and the second reason involves internal gradual erosion of foundation material resulting in the piping phenomenon. Therefore, in all cases which foundation of a structure constructed on permeable bed, it is necessary that bearing pressure on the contact surface between foundation and structure and also output hydraulic gradient in down-level of contact surface to be estimated. For example, such studies about diversion dams located on the bed of alluvial river is crucial, because of upstream water storage and then creating the headwater differential in upstream and downstream of the dam, seepage under the foundation will be occurred and this may cause pressure to the contact surface between foundation and dam.

Different methods have been applied to make economic plans to reduce uplift pressure in the concrete gravity dams, some of the most important of them can be mentioned as the bed filter and cut off walls which are placed on the upstream and downstream of the dam (to increase the flow path length and creating more head loss) and construction of drain on the downstream of the dam or in the appropriate place between the two cut off walls (to reduce output gradient) [2]. Mainly, the present study is aimed to investigate the effects of these parameters to reduce output gradient and uplift pressure. A mathematical model is presented to show the effects of number, location and length of cut off walls and drain on uplift pressure and output hydraulic gradient at key locations under diversion dams and general stability of dam has been evaluated using Mseep program. The fundamental concepts of Mseep program are based on the Finite Element Method while the proposed model applied in this study utilizes the Boundary Element Method.

## 2 Theoretical development

The Laplace equation is the governing equation for two-dimensional potential flows in porous medium which reads as:

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2} = 0$$
 (1)

 $\phi$  in this equation is the potential function and flow velocity components will be derived from this function. Solution of Laplace's equation and determination of the potential function at key locations will result in the specifications of flow. To solve the Laplace's equation different methods have been suggested among them the most important are:

- 1. Laboratory methods constructed based on physical models.
- 2. Graphical methods in which the  $\phi$  values obtained by drawing the flow lines and equipotential lines (flow net).

- 3. Theoretical empirical methods (such as the Lane and Bligh methods).
- 4. Combined potential method which based on complex functions leed to analytical solution of equations.
- 5. Numerical methods such as the Finite differences, Finite elements, and Boundary Elements.

Nowadays, numerical methods for solving Laplace equation are used widely and have become common. High speed in determination of the potential and hydraulic gradient values in all parts of a permeable field is the advantage of these methods. One of the most appropriate methods in solving the Laplace's equation is the Boundary Element Method (BEM), in which the approximate shape functions are used so that within the specific domain satisfied governing differential equation on the field but not at the boundaries [4]. Unknown coefficients of these functions are calculated through applied boundary conditions at certain points at the boundaries. Therefore unknown function values at the boundaries are obtained completely and then with the position of internal domain points, unknown function values of the points can be calculated. A sample of such networking system is shown in fig. 1. Concept of these elements is that the potential function is constant along the elements and equal to the potential value at the center of element. This assumption could not be very unreasonable, if large numbers of elements are selected.



## Figure 1: Schematic networking of mathematical model of the foundation of concrete gravity dam.

The governing equation in BEM can be written in a matrix form as:  $[H]\{\emptyset\} = [G]\{q\}$  (2)

In which:

- {Ø}: Potential vector in elements.
- *{q}*: Velocity vector in elements.
- [*H*] and [*G*]: matrix of coordinates which defined as:



$$H = \int_{\eta} \frac{\partial \phi^*}{\partial r} d\eta \tag{3}$$

$$G = \int_{\eta} \phi^* \, d\eta \tag{4}$$

$$\emptyset^* = \frac{1}{4\pi r} \tag{5}$$

in which:

r: distance from the centre of element.

Solving the above equations will lead us to the magnitudes of  $\emptyset$  in the nodes; and and subsequently the pressure and hydraulic gradient can be derived from the potential function.

Variables in the present model which illustrated in fig. 2 are as follows [3]:

H1 = water head on upstream.

H2 = water head on downstream.

 $\Delta H$  = height of effective water.

T = depth of permeable layer.

 $D_1$  = length of upstream cut off wall.

 $D_2$  = length of downstream cut off wall.

L =length of bed filter.

 $L_1$  = distance from the middle cut off wall to the upstream cut off wall.

 $L_2$  = distance from the drain to the upstream cut off wall.

f = length of drain.

t = thickness of the cut off wall.

t'= thickness of bed filter.





The reasons for applying the boundary elements method to this particular case can be explained as follows:

1. Laplace equation can be easily solved by this method and the results are satisfactory.



2. Due to less degree of freedom in this method rather than other numerical methods (such as the Finite Element Method), the number of equations to be solved in the present method under the similar conditions will be less.

3. In the present method, networking of system is easily possible due to onedimensional nature of elements.

4. Major changes in the system, in this particular case, generally are at the boundaries, thus using the Boundary Element Method (BEM) which analyzed networked boundaries will be simpler than other numerical methods.

Boundary in the present system, including boundaries A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R can be divided into two parts, fig. 1, [1]:

1. Boundaries that have the boundary conditions of the type of  $\phi = \phi$ . These borders include some points in which the potentials are known (e.g. RQ, IH and DC boundaries in the fig. 1 that amount of piezometric heads are equal to values H1, H2 and H3)respectively. It is to be mentioned that if the Laplace equation needs to be considered in the form of  $\nabla^2 h = 0$  (h is the piezometric head), system boundary conditions will be of the type of  $h = \overline{h}$ .

2. Boundaries that have the boundary conditions of the type of  $\frac{\partial \phi}{\partial n} = \overline{q}$ : in this

boundaries flow velocity (derivative of the potential function) is a certain amount (n is unit vector perpendicular to the surface). For example, boundaries of impermeable layer, cut off walls and bed filter in fig. 1, include this type of boundaries (e.g. AB, BC, DE, EF, FG, and GH boundaries and other similar boundaries).

After introduction of the boundaries and sorting system network into boundary elements with certain geometric model and node coordinates, the computer can easily solve boundary element method equations system and find the potential and hydraulic gradient values at specific points. Noting the following comments, [5], in the model networking are necessary:

1. System lateral boundaries should be considered so that the effect of the uplift pressure at those is negligible.

2. Flow velocity at the lateral boundaries is considered to be equal to zero (similar to impermeable layer line AB).

3. At the cut off wall, bed filter and impermeable layers, flow velocity is equal to zero.

4. On the levels of the ground under the bed filter, water head has specific values and is equal to the height of water above lines.

5- Along the drain the piezometric head like the uplift pressure is equal to H2.

6- Elements numbering is conducted in an arbitrary direction (for instance in counter clockwise) in accordance with the order.

7. Capability of the present model is in homogeneous and isotropic soils and in other situations equivalent permeability coefficient should be used.

8. One of the advantages of this model is that, any change in the system boundaries can be reviewed with the introduction of boundary conditions related to it.



9. To evaluate the effect of each variable, other parameters are assumed to be constant, and eventually led to the charts that show the effect of proposed variable on the uplift pressure or output hydraulic gradient.

10. Due to the importance of uplift pressure at point G and the hydraulic gradient at point D, fig. 1, only these two key locations are taken into account.

Using Mseep program, parameters in fig. 2 have been taken into the program as input and ultimately, pressure and potential values have been calculated at any location on the permeable layer under the dam. Then the results have been plotted in some diagrams. The following limitation exists in the modelling procedure:

1. To consider the effect of various parameters, other parameters are assumed to be constant.

2. Due to the low importance of thickness of the cut off wall, it is assumed to be constant.

3. Instead of water heads at upstream and downstream to be considered as two different parameters, the difference of the water head between upstream and downstream (H) is considered as a critical parameter.

4. Maximum hydraulic gradient considered at the end of the dam impermeable bed, because of the risk of piping in that region.

5. Because of the existence of filter, the variation of the uplift pressure is not uniform and in this study the average uplift pressure value is calculated and analyzed.

## **3** Analyses of the results

Analyses of the uplift pressure and the hydraulic gradient output values calculated at key locations on boundaries with the Mseep program and present methods lead to different designed curves. The following results can be obtained from this investigation:

1. Increasing the length of upstream cut off wall (D1) results in decreasing the uplift pressure value at point G and reducing the output hydraulic gradient at point D caused by increased in the flow length along the way. Increasing distance from the drain to the cut off wall causes the output gradient value to rise at point D, but uplift pressure might increase or decrease at point G (most of the curves have the uplift pressure reduction). Increasing the output gradient due to more density of input flow lines on the foundation can be expected when the drain moves to downstream, and due to low density of the flow lines after drain, the uplift pressure will be decreased at point G. The results are consistent with the results of the Mseep program, fig. 3.

2. Placing the filter upstream results in the uplift pressure and hydraulic gradient values to be decreased, but at the vanes of end cut off wall, the uplift pressure as a factor of instability will be increased [6]. Fig. 4 shows the results of Mseep program which confirm the results of the present investigation.





Figure 3: Combined effect of locating filter and length of the primary cut off wall on uplift pressure and hydraulic gradient.



Figure 4: Combined effects of the location of filter and the length of the lateral cut off wall on uplift pressure and hydraulic gradient.

3. Increasing the drain length increase (f) makes a significant decrease in uplift pressure and especially in the output hydraulic gradient, fig. 5.

4. Probability of the piping phenomenon (hydraulic gradient reaches to a critical value) is possible in high effective head (about four times depth of permeable foundation). The occurrence of such a potential is unlikely in a small diversion dam [7]. In other word, it can be concluded that in small diversion dams, the hydraulic gradient cannot be an effective design parameter because in the normal cases it never reaches to the critical limit. Fig. 6 shows the results of the Mseep program which confirms the mentioned result.



Figure 5: Effect of filter width on the uplift pressure and hydraulic gradient.



Figure 6: Probability of the piping phenomenon in small concrete diversion dam with a filter.

## 4 Summary and conclusions

The results obtained from the present model and comparison with those obtained from Mseep program indicates the following conclusions:

1. The cut off wall located upstream reduces the uplift pressure and output hydraulic gradient; both of them are desirable for designers and suitable for stability of the dam.



2. The cut off wall located downstream reduces the output hydraulic gradient; but it increases the uplift pressure which is considered as a negative factor on stability.

3. Filter (drain), reduces the uplift pressure and output hydraulic gradient. Reducing pressure is considerably more where the filter is located upstream of dam.

4. In small diversion dams the hydraulic gradient is less than its critical value and it is not considered as a major parameter in design.

5. Filter, itself, does all the effective functions of cut off wall without the negative effects. This effect will increase where filter is close to the upstream of dam and in this case, filter does the functions of the end cut off wall, and if the output discharge does not create any limit for designer, the end cut off wall can be shortened or even eliminated.

By comparing the results of the present model and Mseep program, it can be concluded that the present method introduces faster and easier approach to determine the output hydraulic gradient and uplift pressure with simple assumptions and solving few equations. An economical plan could be achieved for the optimized design of the foundation of diversion dams that minimizes the total cost of project with the results obtained from the proposed mathematical model. Dimensions of the cut off wall and drain will be designed to minimize the output hydraulic gradient and uplift pressure of dam foundation, with reduced concrete to the lowest volume as much as it is possible.

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