Measures for the postprocessing of grounding electrodes transient response

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Abstract

Various measures for quantifying the transient response of simple grounding systems are proposed in this paper. In addition to the standard transient impedance concept the suggested measures arising from the circuit theory are instantaneous power, average power and total energy stored in the near field of a grounding electrode. The frequency response of the grounding electrode is obtained by using the antenna model (AM) while the associated transient response is computed using the Inverse Fourier Transform. The integro-differential realtionships arising from the wire antenna theory are numerically handled via the Galerkin-Bubnov scheme of the Indirect Boundary Element Method (GB-IBEM). A number of illustrative numerical results are presented in the paper.

1 Introduction

Transient analysis of grounding systems, important for protection of personnel and equipment, is of widespread interest in electromagnetic compatibility (EMC) and high voltage (HV) engineering. Transient modeling of grounding systems can be carried out applying either the transmission line model (TLM) [1–3] or antenna (electromagnetic) model (AM) [4–6]. An important parameters arising from studies of transients in grounding systems is the transient impedance. Further to the transient impedance concept for postprocessing transient responses, widely adopted within EMC community, this work deals with some additional measures of a horizontal electrode transient response. These measures arise from the circuit theory and are, as follows: instantaneous power, average power and the total energy accumulated in the near field of the electrode.

WIT Transactions on Modelling and Simulation, Vol 44,
www.witpress.com, ISSN 1743-355X (on-line)
doi:10.2495/BE070291

The antenna model of the electrode used in in this work is based on the Pocklington integro-differential equation approach [4–6]. The current distribution along the horizontal grounding electrode is governed by the Pocklington integro-differential equation which is solved via the Galerkin-Bubnov scheme of the Indirect Boundary Element Method (GB-IBEM) [12, 13]. The effect of a dissipative half-space is taken into account via the corresponding reflection coefficient thus avoiding the solution of analytically demanding and numerically time consuming Sommerfeld integrals. The voltage at the feed point is obtaned by analytically integrating the normal electric field from the electrode surface to infinity. The input impedance of the electrode (transfer function of the system) is computed as a ratio of evaluated voltage and the feed point current.

The frequency response of the horizontal electrode to a particular current source excitation is obtained multiplying the input impedance spectrum with Fourier transform of the actual lightning current waveform. The transient response of the horizontal grounding electrode is assessed applying the inverse Fourier transform. Once determining the transient response of the electrode, the transient behaviour can be quantified using the standard concept of transient impedance and also by using the proposed circuit theory measures.

2 Equivalent antenna model of the grounding electrode

The geometry of interest, shown in Fig 1, is the horizontal grounding electrode of length L and radius a, buried in a lossy medium at depth d and excited by a current source. The wire is assumed to be perfectly conducting and its dimensions satisfy the thin wire approximation (TWA) conditions [12].

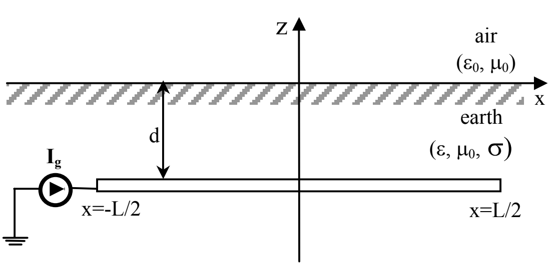

Figure 1: Horizontal grounding wire energized by a current generator I_g.

The Pocklington integro-differential equation for the current distribution along the horizontal grounding electrode can be derived by expressing the electric field in terms of the Hertz vector potential and by satisfying the given boundary conditions for the tangential field components at the electrode surface [4–6].

2.1 Thin wire integral equation for a horizontal electrode

The current induced along the horizontal grounding electrode is governed by the Pocklington integro-differential equation [6]:

$$E_x^{exc,H} = -\frac{1}{j4\pi\omega\varepsilon_{eff}} \int_{-L/2}^{L/2} \left[\frac{\partial^2}{\partial x^2} + k_1^2 \right] [g_0^H(x,x') + \Gamma g_i^H(x,x')] I(x')dx' \tag{1}$$

where $I(x')$ is the unknown current distribution along the wire, $E_x^{exc,H}$ is the excitation function, $g_0^H(x,x',z)$ denotes the free space Green function of the form:

$$g_0^H(x,x',z) = \frac{e^{-jk_1 R_{1h}}}{R_{1h}} \tag{2}$$

while $g_i^H(x,x',z)$ arises from the image theory and is given by:

$$g_i^H(x,x',z) = \frac{e^{-k_2 R_{2h}}}{R_{2h}} \tag{3}$$

where R_{1h} and R_{2h} are the distances from the horizontal wire in the lossy ground and from its image in the air to the observation point in the lower medium, respectively. Furthermore, k_1 is the phase constant of a lossy ground:

$$k_1^2 = -\omega^2 \mu \varepsilon_{eff} \tag{4}$$

and ε_{eff} denotes the complex permitivity of the lossy ground:

$$\varepsilon_{eff} = \varepsilon_r \varepsilon_0 - j\frac{\sigma}{\omega} \tag{5}$$

where and ε_r and σ are relative permitivity and conductivity of the ground respectively, and ω denotes the operating frequency.

The presence of a lossy medium is taken into-account via the reflection coefficient while Γ is the corresponding reflection coefficient for the TM polarization [6]:

$$\Gamma = \frac{\frac{1}{\underline{n}}\cos\theta - \sqrt{\frac{1}{\underline{n}} - \sin^2\theta}}{\frac{1}{\underline{n}}\cos\theta + \sqrt{\frac{1}{\underline{n}} - \sin^2\theta}} \tag{6}$$

where θ and $\underline{n}$ are given by:

$$\theta = arctg\frac{|x - x'|}{2d}; \qquad \underline{n} = \frac{\varepsilon_{eff}}{\varepsilon_0} \tag{7}$$

The rigorous AM approach leads to the repeated evaluation of Sommerefeld integrals representing rather time consuming task.

This work features the reflection coefficient (RC) approach [6, 11]. The main advantage of RC approach versus rigorous Sommerfeld integral approach is a simplicity of the formulation and significantly less computational cost. It is worth noting that the RC approach produces results roughly within 10% of these obtained via rigorous Sommerfeld integral approach [11, 14].

2.2 The current source excitation

Within the analysis of the grounding electrodes it is not possible to define the excitation function in the form of an electric field. The horizontal grounding electrode is energized by the injection of a corresponding current pulse represented by an ideal current source with one terminal connected to the grounding electrode and the other one grounded at infinity, as shown in Fig 1.

Consequently:

$$E_x^{exc} = 0 \tag{8}$$

and the Pocklington integro-differential equation (9) becomes homogeneous [4–6].

This current source is included into the integro-differential equation formulation through the following boundary condition:

$$I(-L/2) = I_g \tag{9}$$

where I_g denotes the impressed unit current generator.

3 Boundary element procedure

Solving the integral equation (1) via the GB-IBEM the equivalent current distribution along the horizontal grounding electrode is obtained. The numerical solution steps are outlined below.

Performing certain mathematical manipulations and boundary element discretization the solution for the unknown current $I^e(x)$ along the wire segment can then be written as:

$$I^e(x') = \{f\}^T \{I\} \tag{10}$$

Assembling the contributions from each element the integro-differential equation (9) is transferred into the following matrix equation:

$$\sum_{i=1}^{M} [Z]_{ji} \{I\}_i = 0, \quad \text{and} \quad j = 1,2,...,M \tag{11}$$

www.witpress.com, ISSN 1743-355X (on-line)

where M is the total number of wire segments and $[Z]_{ji}$ is the mutual impedance matrix representing the interaction of the *i*-th source boundary element with the *j*-th observation boundary element:

$$[Z]_{ji} = -\frac{1}{j4\pi\omega\varepsilon_{eff}}\left\{\int_{\Delta l_j}\int_{\Delta l_i}\{D\}_j\{D'\}_i^T g^H(x,x')dx'dx + k^2\int_{\Delta l_j}\int_{\Delta l_i}\{f\}_j\{f'\}_i^T g^H(x,x')dx'dx\right\} \tag{12}$$

Matrices $\{f\}$ and $\{f'\}$ contain the shape functions while $\{D\}$ and $\{D'\}$ contain their derivatives, M is the total number of finite elements, and Δl_i, Δl_j are the widths of *i*-th and *j*-th boundary elements.

The linear approximation over a boundary element is used in this work as it has been shown that this choice provides accurate and stable results for various wire configurations [12, 13].

The excitation function in the form of the current source I_g is taken into account as a forced boundary condition at the first node of the solution vector, i.e.:

$$I_1 = I_g \quad \text{and} \quad I_g = 1e^{j0} \tag{13}$$

providing the linear equation system to be solved properly.

4 The assessment of the transient response

Transient voltage at the feed point (x=-L/2) can be obtained from the convolution integral:

$$v(x,t)\big|_{x=-L/2} = \int_0^t z_{in}(x,t)\big|_{x=-L/2}\, i(x,t-\tau)\big|_{x=-L/2}\, d\tau \tag{14}$$

The frequency response of the grounding system: is obtained by multiplying the frequency spectrum of the excitation function *I*(*f*) with the frequency domain counterpart of the impulse response, i.e. the input impedance spectrum ($Z_{in}f$)):

$$V(f) = I(f)Z_{in}(f) \tag{15}$$

This injected current waveform, i.e. the lightning channel current is given by:

$$i(t) = I_0 \cdot (e^{-\alpha t} - e^{-\beta t}), \quad t \geq 0 \tag{16}$$

where pulse rise time is shaped by constants α and β, while I_0 is the amplitude of the current waveform.

The Fourier transform of the excitation function (26) is given by [12]:

$$I(f) = I_0 \cdot \left(\frac{1}{\alpha + j2\pi f} - \frac{1}{\beta + j2\pi f}\right) \tag{17}$$

Instead of solving convolution integral, as the time domain waveform of the input impedance (the impulse response of the system) is not known, the transient voltage is computed by applying the Inverse Fourier Transform (IFT).

A time domain voltage counterpart, i.e. the IFT of the function $V(f)$ is defined by the integral [16]:

$$v(t) = \int_{-\infty}^{\infty} V(f) e^{j2\pi ft} d\omega \tag{18}$$

as the frequency response $V(f)$ is represented by a discrete set of values the integral (18) cannot be evaluated analytically and the Discrete Fourier transform, in this case the Inverse Fast Fourier Transform (IFFT) algorithm, is used, i.e.:

$$v(t) = IFFT\left[V(f)\right] \tag{19}$$

Implementation of this algorithm generates an error due to discretization and truncation of unlimited frequency spectrum. The discrete set of time domain voltage values is given by [16]:

$$v(n\Delta t) = F \cdot \sum_{k=0}^{N-1} V(k\Delta f) e^{jk\Delta fn\Delta t} \tag{20}$$

where F denotes the highest frequency taken into account, N is the total number of frequency samples, Δf is sampling interval and Δt is the time step.

Therefore, the grounding electrode problem is related to the assessment of the input impedance.

The input impedance is given by the ratio:

$$Z_{in} = \frac{V_g}{I_g} \tag{21}$$

where V_g and I_g are the values of the voltage and the current at the driving point.

The feed-point voltage due to the unit current source can be calculated by integrating the normal electric field component from the electrode surface to infinity, i.e.:

$$V_g = -\int_{\infty}^{a} \vec{E} d\vec{s} \tag{22}$$

Repeating this procedure in the wide frequency band gives the input impedance spectrum.

For the particular case of horizontal grounding electrode integral (22) becomes:

$$V_g = -\int_{\infty}^{a} E_z^H \left(x, z\right) dz \tag{23}$$

where E_z^H is the radial electric field component, normal to the electrode:

$$E_z^H(x,z) = \frac{1}{j4\pi\omega\varepsilon_{eff}} \int_{-L/2}^{L/2} I\left(x'\right) \frac{\partial^2 G^H(x,x',z)}{\partial x \partial z} dx' \tag{24}$$

Inserting equation (24) into (23) and performing some mathematical manipulation [6] yields:

$$V_g = \frac{1}{j4\pi\omega\varepsilon_{eff}} [I\left(-L/2\right) G^H\left(x,-L/2,z\right) - \int_{-L/2}^{L/2} \frac{\partial I\left(x'\right)}{\partial x'} G^H\left(x,x',z\right) dx'] \Big|_{z=a}^{z=\infty} \tag{25}$$

and the input impedance of the grounding wire is determined by the relation:

$$Z_{in} = \frac{1}{j4\pi\omega\varepsilon_{eff} I_g} [I\left(-L/2\right) G^H(x,-L/2,z) - \int_{-L/2}^{L/2} \frac{\partial I(x')}{\partial x'} G^H\left(x,x',z\right) dx'] \Big|_{z=a}^{z=\infty} \tag{26}$$

and the desired impedance spectrum can be computed.

5 Measures for postprocessing the transient response

The well-established measure for analyzing the transient behaviour of the horizontal grounding electrode is the transient impedance which is defined as a ratio of transient voltage and current at the input terminals [4]:

$$z(x,t)\big|_{x=-L/2} = \frac{v(x,t)}{i(x,t)}\Big|_{x=-L/2} \tag{27}$$

where $i(t)$ is the current injected at an end of the horizontal electrode, Fig. 1.

Once obtaining the transient voltage flowing through the horizontal electrode it is possible to calculate certain parameters providing additional measures of this transient response. Such parameters can be found in the theory of electric circuits.

The convenient parameters quantifying the horizontal electrode transient response, suggested in this work, are: instantaneous power and the total energy accumulated in the near field of the electrode.

According to the theory of electric circuits the amount of delivered power strongly depends on the particular waveform. Thus, a time varying current delivers an average power to a grounding electrode and it is given by the product:

$$p(x,t)\big|_{x=-L/2} = u(x,t) \cdot i(x,t)\big|_{x=-L/2} \tag{28}$$

The corresponding average power P_{av} is determined by the integral relation:

$$P_{av} = \frac{1}{T_0}\int_0^{T_0} p(x,t)\Big|_{x=-L/2}\,dt = \frac{1}{T_0}\int_0^{T_0} u(x,t)\cdot i(x,t)\Big|_{x=-L/2}\,dt \tag{29}$$

In accordance to the circuit theory, the energy in the near field of the grounding electrode can be determined by temporally integrating the instantaneous power:

$$W_{tot}(x,t)\Big|_{x=-L/2} = \int_0^t p(x,t)\Big|_{x=-L/2}\,dt \tag{30}$$

Total stored energy can be obtained by specifying $t=T_0$.

6 Computational example

Numerical results shown in Figures 2 to 9 are related to the grounding electrode of radius a=5mm buried at depth d=5m in a lossy medium (σ=0.1mS/m, ε_r=10). Figures 2 to 9 show transient voltage, transient impedance, instantaneous power and energy accumulated in the grounding electrode near field within the considered time interval of 10μs for the various set of parameters. Analyzing the proposed energy measures it is obvious that the transient response of the electrode is particularly dependent on its length.

7 Concluding remarks

Some measures for postprocessing of the horizontal grounding electrode transient response are presented in this work. The transient response of the electrode is obtained using the frequency domain antenna model of the grounding electrode and the Inverse Fourier Transform. The integral relationships arising from the wire antenna model are numerically treated by using the GB-IBEM.

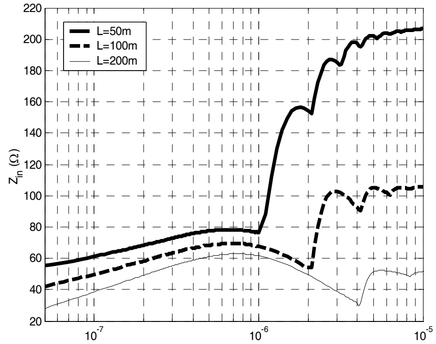

Figure 2: Transient impedance (d=5m, σ=0.0001S/m).

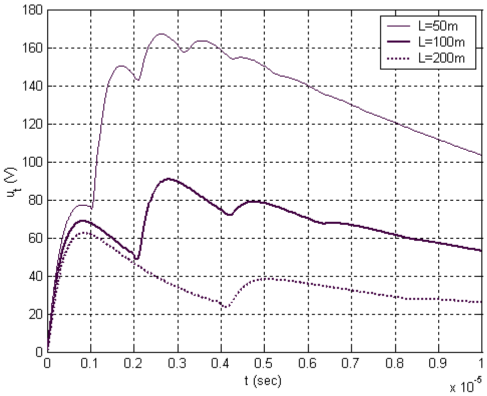

Figure 3: Transient voltage for various wire lengths (d=5m, σ=0.0001S/m).

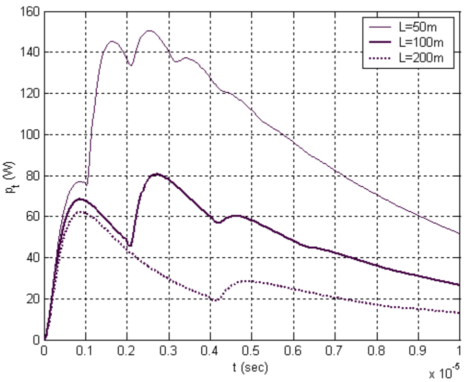

Figure 4: Instantaneous power for various wire lengths (d=5m, σ=0.0001S/m)

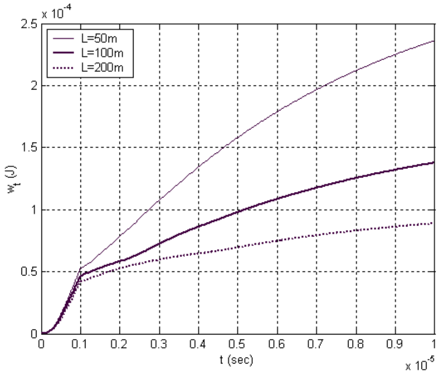

Figure 5: Total energy for various wire lengths (d=5m, σ=0.0001S/m).

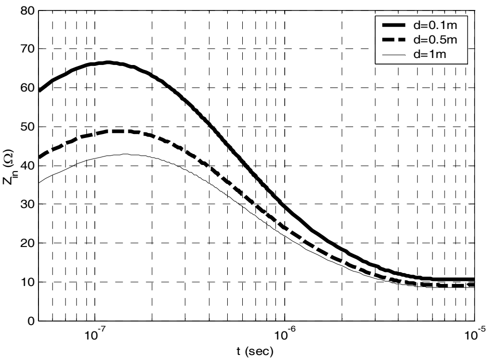

Figure 6: Transient impedance (L=100m, σ=0.001S/m).

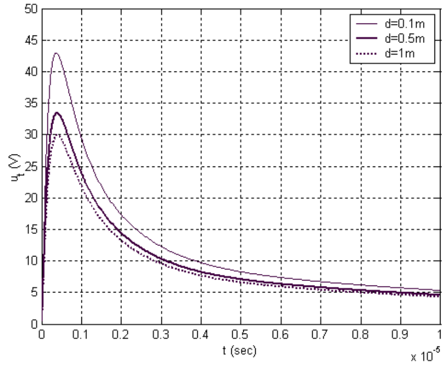

Figure 7: Transient voltage for various burial depths (L=100m, σ=0.001S/m).

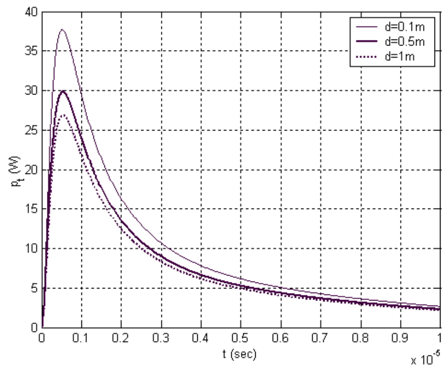

Figure 8: Instantaneous power for various burial depths (L=100m, σ=0.001S/m).

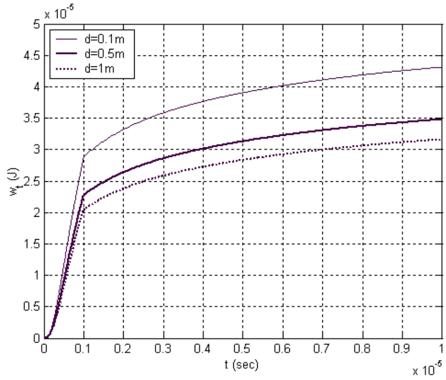

Figure 9: Total energy for various burial depths (L=100m, σ=0.001S/m).

Further to the standard transient impedance concept some additional measures for the transient response are presented in this paper. Once obtaining the transient response of the electrode it is possible to calculate the measures for quantifying the transient response in terms of the average power, instantaneous power and total energy stored in the electrode near field.

Further extension of the present analysis will involve the treatment of complex grounding systems including interconnected conductors.

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