Linear analysis of building floor structures by a BEM formulation based on Reissner's theory

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Abstract

In this work, the plate bending formulation of the boundary element method (BEM) based on the Reissner's hypothesis is extended to the analysis of zoned plates in order to model a building floor structure. In the proposed formulation each sub-region defines a beam or a slab and depending on the way the sub-regions are represented, one can have two different types of analysis. In the simple bending problem all sub-regions are defined by their middle surface. On the other hand, for the coupled stretching–bending problem all sub-regions are referred to a chosen reference surface, therefore eccentricity effects are taken into account. Equilibrium and compatibility conditions are automatically imposed by the integral equations, which treat this composed structure as a single body. The bending and stretching values defined on the interfaces are approximated along the beam width, reducing therefore the number of degrees of freedom. Then, in the proposed model the set of equations is written in terms of the problem values on the beam axis and on the external boundary without beams. Finally some numerical examples are presented to show the accuracy of the proposed model.
Keywords: plate bending, boundary elements, building floor structures.

1 Introduction

The boundary element method (BEM) has already proved to be a suitable numerical tool to deal with plate bending problems. The method is particularly recommended to evaluate internal force concentrations due to loads distributed over small regions that very often appear in practical problems. Moreover, the same order of errors is expected when computing deflections, slopes, moments

WIT Transactions on Modelling and Simulation, Vol 44, © 2007 WIT Press
www.witpress.com, ISSN 1743-355X (on-line)
doi:10.2495/BE070221

and shear forces. They are not obtained by differentiating approximation function as for other numerical techniques.

Several models to analyze plate reinforced by beams, using BEM coupled with the finite element method (FEM), have already been proposed (see Hu and Hartley [1], Tanaka. and Bercin [2], Sapountzakis and Katsikadelis [3]). In those works the BEM and FEM approximate, respectively, plate and beam elements. However, for complex floor structures the number of degrees of freedom may increase rapidly diminishing the solution accuracy.

An alternative scheme to reduce the number of degrees of freedom has been recently proposed by Fernandes and Venturini in [4] and [5] using only a BEM formulation based on Kirchhoff's hypothesis, where the building floor is modeled by a zoned plate. In the first work is proposed a formulation to perform simple bending analysis where the tractions are eliminated along the interfaces. Moreover in order to reduce the number of degrees of freedom some Kinematics assumptions were made along the beam width. In the second work this formulation is extended to take into account the membrane effects which are associated with bending due to the relative positions of the structural elements.

In this work the BEM formulation developed in [5] is modified to take into account the Reissner's hypothesis instead of the Kirchhoff's. In the proposed model the tractions related to the bending problem is no longer eliminated on the interfaces. Therefore traction and displacements related to both problems (bending and stretching) are approximated along the beam width, leading to a model where the problem values are defined only on the beams axis and on the plate boundary without beams. The accuracy of the proposed model is illustrated by numerical examples whose analytical results are known.

Note that in the Kirchhoff's theory (see Fernandes and Venturini [5], Hartmann and Zotemantel [6] and Kirchhoff [7]) are defined only four boundary values and its inaccuracy turns out to be important for thick plates, especially in the edge zone of the plate and around holes whose diameter is not larger than the plate thickness. In the Reissner's theory (see Reissner [8], Weën [9] and Palermo [10]) which can be used either for thin or thick plates, are defined six boundary values and it is more accurate because it takes into account the shear deformation effect.

2 Basic equations

Without loss of generality, let us consider the plate depicted in figure 1(a), where t_1, t_2 and t_3 are the thicknesses of the sub-regions Ω_1, Ω_2 and Ω_3, whose external boundaries are Γ_1, Γ_2 and Γ_3, respectively. The total external boundary is given by Γ while Γ_{jk} represents the interface between the adjacent sub-regions Ω_j and Ω_k.. In the simple bending analysis all sub-regions are represented by their middle surface, as shown in figure 1(c), while for the coupled stretching-bending problem the Cartesian system of co-ordinates (axes x_1, x_2 and x_3) is defined on a chosen reference surface (see figure 1(b)), whose distance to the sub-regions middle surfaces are given by c_1, c_2 and c_3. As in figure 1(b) the reference surface is adopted coincident to Ω_2 middle surface one has $c_2=0$.

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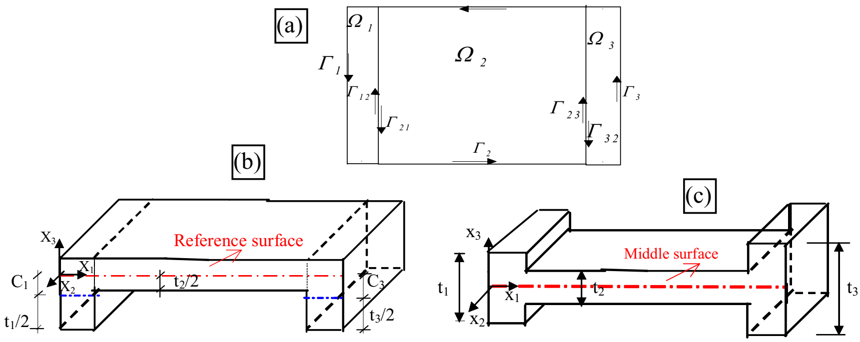

Figure 1: (a) Reinforced plate; (b) reference surface view, (c) middle surface view.

Let us consider initially, the bending problem. For a point placed at any of those plate sub-regions, the following equations are defined:

-The equilibrium equations in terms of internal forces:

$$M_{ij},_j - Q_i = 0 \qquad \text{i, j =1, 2} \tag{1}$$

$$Q_i,_i + g = 0 \tag{2}$$

where g is the distributed load acting on the plate middle surface, m_{ij} are bending and twisting moments and Q_i represents shear forces.

-The generalised internal forces written in terms of displacement:

$$M_{ij} = \frac{D(1-\nu)}{2}\left(\phi_i,_j + \phi_j,_i + \frac{2\nu}{1-\nu}\phi_k,_k\,\delta_{ij}\right) + \frac{\nu g}{(1-\nu)\lambda^2}\delta_{ij} \tag{3}$$

$$Q_i = \frac{D(1-\nu)}{2}\lambda^2\left(\phi_i + w,_i\right) \qquad \text{i, j =1, 2;} \tag{4}$$

where ϕ_i is the rotation in the i direction, w the deflection, $D=Et^3/(1-\nu^2)$ the flexural rigidity, ν the Poisson's ration, λ a constant related to shear effect given by $\lambda = \sqrt{10}\,/\,t$ and δ_{ij} is the Kronecker delta.

-Finally, the plate bending differential equations given by:

$$Q_i - \frac{1}{\lambda^2}\nabla^2 Q_i + \frac{1}{(1-\nu)\lambda^2}\frac{\partial g}{\partial x_i} = -D\frac{\partial}{\partial x_i}\nabla^2 w \qquad \text{i=1, 2} \tag{5}$$

$$\nabla^4 w = \frac{1}{D}\left[g - \frac{(2-\nu)}{(1-\nu)\lambda^2}\nabla^2 g\right] \tag{6}$$

where $w,_{iijj} = \nabla^4 w$, being ∇^4 the bi-harmonic operator; $w,_{ii} = \nabla^2 w$ being ∇^2 the bi-Laplacian operator.

Equations (5) and (6) result into the set of differential equations, being eqns. (5) and. (6) a second and fourth order equation, respectively, leading therefore to six independent boundary values: M_n, M_{ns}, Q_n, w, ϕ_n and ϕ_s being (n, s) the local co-ordinate system, with n and s referred to the plate boundary normal and tangential directions, respectively.

Considering now the stretching problem, the in-plane equilibrium equation is:

$$N_{ij},_j + b_i = 0 \tag{7}$$

where b_i are body forces distributed over the plate middle surface and N_{ij} is the membrane internal force, which, for plane stress conditions, can be written in terms of the in-plane displacements u_i derivatives as follow:

$$N_{ij} = Gt\left[\frac{2\nu}{(1-\nu)}u_k,_k\,\delta_{ij} + (u_i,_j + u_j,_i)\right] \tag{8}$$

The problem definition is then completed by assuming the following boundary conditions over Γ: $U_i = \overline{U}_i$ on Γ_u (generalised displacements: deflections, rotations and in-plane displacements) and $P_i = \overline{P}_i$ on Γ_p (generalised tractions: bending and twisting moments, shear forces and in-plane tractions), where $\Gamma_u \cup \Gamma_p = \Gamma$. Note that the in-plane displacements and tractions are considered only for the coupled stretching-bending problem.

3 Integral representations

For the simple bending problem the following weighted residual equation can be written for a simple plate:

$$\int_{\Omega}\left[\phi^*_{ki}\left(\mathrm{M}_{ij},_j - \mathrm{Q}_i\right) + \left(\mathrm{Q}_i,_i + \mathrm{g}\right)\mathrm{w}^*_k\right]d\Omega = \int_{\Gamma_u}\left[\left(\overline{\phi}_i - \phi_i\right)\mathrm{M}^*_{ki} + \left(\overline{\mathrm{w}} - \mathrm{w}\right)\mathrm{Q}^*_{kn}\right]d\Gamma$$
$$- \int_{\Gamma_P}\left[\left(\overline{M}_i - M_i\right)\phi^*_{ki} + \left(\overline{Q}_n - Q_n\right)w^*_k\right]d\Gamma \quad \text{i, j =1, 2; k=1, 2, 3} \tag{9}$$

where the superscript * refers to the fundamental problem; k is the fundamental load direction with k = 1, 2 defining unit moments applied in the x_1 and x_2 directions and k=3 is related to a unit load acting in the x_3 direction.

Integrating (9) by parts twice, considering eqns (3) and (4) and writing the values in terms of the local system of coordinates (n,s), the integral equation of the generalised displacements can be obtained:

$$c(q)U_k(q) = \int_{\Omega_g} g\left[w^*_k - \frac{\nu}{(1-\nu)\lambda^2}\phi^*_{ki},_i\right]d\Omega - \int_{\Gamma}\left[\phi_n M^*_{kn} + \phi_s M^*_{kns} + wQ^*_{kn}\right]d\Gamma +$$
$$+ \int_{\Gamma}\left[M_n\phi^*_{kn} + M_{ns}\phi^*_{ks} + Q_n w^*_k\right]d\Gamma \qquad \text{k=m, l, 3;} \qquad \text{i=1, 2} \tag{10}$$

where q is the collocation point, Ω_g the area where the load g is distributed, $c(q)$ is the free term given by: $c(q) = 0$, $c(q) = 1$ and $c(Q) = 1/2$, respectively, for external, internal and boundary points; $U_m = \phi_m$ $U_l = \phi_l$ and $U_3 = w$, being m and l the local system (n, s) for boundary points or any direction for internal points.

For a zoned plate, as the one depicted in the figure 1, eqn. (10) is valid to each sub-region separately. Then, taking into account the equilibrium and compatibility conditions, writing eqn. (10) to all sub-regions and summing them one can write the integral representation for the simple bending problem:

$$U_k(q) = \sum_{j=1}^{N_s} \int_{\Omega_g} g\left[w_k^{*j} - \frac{\nu}{(1-\nu)\lambda^2} \phi_{ki}^{*j},_i \right] d\Omega - \sum_{j=1}^{N_s} \int_{\Gamma_1} \left[\phi_n M_{kn}^{*j} + \phi_s M_{kns}^{*j} + w Q_{kn}^{*j} \right] d\Gamma$$
$$- \sum_{j=1}^{N_{\text{int}}} \int_{\Gamma_{ja}} \left\{ \phi_n \left[M_{kn}^{*j} - M_{kn}^{*a} \right] + \phi_s \left[M_{kns}^{*j} - M_{kns}^{*a} \right] + w \left[Q_{kn}^{*j} - Q_{kn}^{*a} \right] \right\} d\Gamma$$
$$+ \sum_{j=1}^{N_s} \int_{\Gamma_1} \left[M_n \phi_{kn}^{*j} + M_{ns} \phi_{ks}^{*j} + Q_n w_k^{*j} \right] d\Gamma$$
$$+ \sum_{j=1}^{N_{\text{int}}} \int_{\Gamma_{ja}} \left\{ M_n \left[\phi_{kn}^{*j} - \phi_{kn}^{*a} \right] + M_{ns} \left[\phi_{ks}^{*j} - \phi_{ks}^{*a} \right] + Q_n \left[w_k^{*1} - w_k^{*a} \right] \right\} d\Gamma \qquad (11)$$

where N_s and N_{int} are the sub-regions and interfaces number, Γ_{ja} represents a interface for which the subscript a denotes the adjacent sub-region to Ω_j

The bending equation for the coupled stretching-bending problem is obtained from eqn. (11) by writing the moment values on the Ω_j middle surface in terms of their values on the reference surface ($\boldsymbol{M_n^r}$ and M_{ns}^r), as follow:

$$\boldsymbol{M_n^j} = \boldsymbol{M_n^r} + \boldsymbol{p_n c_j} \qquad (12)$$

$$\boldsymbol{M_{ns}^j} = \boldsymbol{M_{ns}^r} + \boldsymbol{p_s c_j} \qquad (13)$$

where p_n and p_s are the in-plane tractions.

Then the bending integral equation for the coupled problem, where all values are referred to the reference surface, reads:

$$U_k(q) = \sum_{j=1}^{N_s} \int_{\Omega_g} g\left[w_k^{*j} - \frac{\nu}{(1-\nu)\lambda^2} \phi_{ki}^{*j},_i \right] d\Omega$$
$$- \sum_{j=1}^{N_s} \int_{\Gamma_1} \left[\phi_n M_{kn}^{*j} + \phi_s M_{kns}^{*j} + w Q_{kn}^{*j} \right] d\Gamma + \sum_{j=1}^{N_s} \int_{\Gamma_1} \left[M_n \phi_{kn}^{*j} + M_{ns} \phi_{ks}^{*j} + Q_n w_k^{*j} \right] d\Gamma$$
$$- \sum_{j=1}^{N_{int}} \int_{\Gamma_{ja}} \left\{ \phi_n \left[M_{kn}^{*j} - M_{kn}^{*a} \right] + \phi_s \left[M_{kns}^{*j} - M_{kns}^{*a} \right] + w \left[Q_{kn}^{*j} - Q_{kn}^{*a} \right] \right\} d\Gamma$$

$$+\sum_{j=1}^{N_{int}}\int_{\Gamma_{ja}}\left\{M_n\left[\phi_{kn}^{*j}-\phi_{kn}^{*a}\right]+M_{ns}\left[\phi_{ks}^{*j}-\phi_{ks}^{*a}\right]+Q_n\left[w_k^{*1}-w_k^{*a}\right]\right\}d\Gamma$$

$$+\sum_{j=1}^{N_{int}}\int_{\Gamma_{ja}}\left\{p_n\left[c_j\phi_{kn}^{*j}-c_a\phi_{kn}^{*a}\right]+p_s\left[c_j\phi_{ks}^{*j}-c_a\phi_{ks}^{*a}\right]\right\}d\Gamma+\sum_{j=1}^{N_s}\int_{\Gamma_l}c_j\left[p_n\phi_{kn}^{*j}+p_s\phi_{ks}^{*j}\right]d\Gamma \quad (14)$$

Let us now consider the stretching problem. For simplicity and also to eliminate the in-plane tractions along the interfaces, the fundamental value $u_{ki}^{*(j)}$ related to Ω_j will be written in terms of u_{ki}^{*} referred to the sub-region where the collocation point is placed as follow:

$$u_{ki}^{*(j)}=u_{ki}^{*}\,\overline{E}/\overline{E}_j \quad (15)$$

where $\overline{E}_j=E_jt_j$.

From the weighted residual method and considering eqn. (15) one can derive the integral representation of displacements for one sub-region. The integral equation for a zoned plate is obtained by summing the equations of all sub-regions and enforcing equilibrium and compatibility conditions along interfaces. Moreover for the coupled problem the in-plane displacements defined over the middle surface (u_s and u_n) have to be written in terms of their values on the reference surface ($u_i^j=u_i^r-c_j\phi_i$, with i=n,s). Then the following stretching integral equation for the coupled stretching-bending problem can be obtained:

$$\left[-c(q)\phi_k(q)+u_k(q)\right]=-\sum_{i=1}^{N_S}\frac{\overline{E}_i}{\overline{E}}\int_{\Gamma}\left[u_np_{kn}^{*}+u_sp_{ks}^{*}\right]d\Gamma+$$

$$-\sum_{m=1}^{N_{\rm int}}\frac{\left(\overline{E}_j-\overline{E}_a\right)}{\overline{E}}\int_{\Gamma_{ja}}\left[u_np_{kn}^{*}+u_sp_{ks}^{*}\right]d\Gamma_{ja}+\sum_{i=1}^{N_S}\frac{\overline{E}_i}{\overline{E}}\int_{\Gamma}c_i\left[p_{kn}^{*}\phi_n+\boldsymbol{p_{ks}^{*}}\phi_s\right]d\Gamma+$$

$$\sum_{m=1}^{N_{\rm int}}\frac{\left(\overline{E}_jc_j-\overline{E}_ac_a\right)}{\overline{E}}\int_{\Gamma_{ja}}\left(p_{kn}^{*}\phi_n+p_{ks}^{*}\phi_s\right)d\Gamma+\int_{\Gamma}\left(u_{kn}^{*}p_n+u_{ks}^{*}p_s\right)d\Gamma+\int_{\Omega_i}\left(u_{kn}^{*}b_n+u_{ks}^{*}b_s\right)d\Omega \quad (16)$$

Note that in eqn (16) the in-plane tractions were eliminated from the interfaces, where the only remaining values are the displacements.

Let us now consider the beam B_3 represented in figure 2(a) by the sub-region Ω_3. In order to reduce the number of degrees of freedom, the displacements w, u_s, u_n, ϕ_s and ϕ_n will be assumed to be linear along the beam width, leading to a model where the values are defined along the beam skeleton line instead of its boundary. Thus the displacement related to the beam interfaces are translated to the skeleton line, as follows:

$$\phi_k^{\Gamma_{32}}=\phi_k+\phi_{k,n}\,b_3/2 \qquad \text{k=n,s} \quad (17a)$$

$$\phi_k^{\Gamma_{31}}=-\left[\phi_k-\phi_{k,n}\,b_3/2\right] \quad (17b)$$

$$w^{\Gamma_{32}} = w + w,_n b_3 / 2 \tag{17c}$$

$$w^{\Gamma_{31}} = w - w,_n b_3 / 2 \tag{17d}$$

$$u_k^{\Gamma_{32}} = u_k + u_k,_n b_3 / 2 \tag{18a}$$

$$u_k^{\Gamma_{31}} = -\left[u_k - u_k,_n b_3 / 2\right] \tag{18b}$$

where b_3 is the beam width , $\phi_k^{\Gamma_{ij}}$, $u_k^{\Gamma_{ij}}$ and $w^{\Gamma_{ij}}$ are displacement components along the interface Γ_{ij}; ϕ_k , w, $\phi_k,_n$, $u_k,_n$ and $w,_n$ are skeleton line components.

Observe that adopting the approximations defined in eqns (17) and (18), new variables related to the beam axis appear in the formulation: the rotations $w,_n$ $u_s,_n$ and $u_n,_n$ and the curvatures $\phi_s,_n$ and $\phi_n,_n$ whose integral representations can be easily obtained by differentiating eqns. (11), (14) or (16).

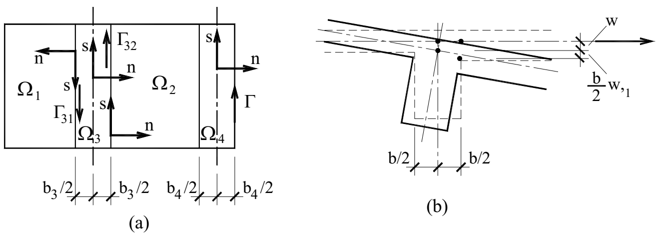

Figure 2: (a) reinforced plate view; (b) deflection approximations along interfaces.

The tractions defined on the interfaces will be written in terms of its components along the beam axis as follows:

$$Q_n^{\Gamma_{32}} = -Q_n^{\Gamma_{31}} = Q_n \tag{19}$$

$$M_n^{\Gamma_{32}} = M_n - Q_n b_3 / 2 \tag{20}$$

$$M_n^{\Gamma_{31}} = M_n + Q_n b_3 / 2 \tag{21}$$

$$M_{ns}^{\Gamma_{32}} = M_{ns}^{\Gamma_{31}} = M_{ns} \tag{22}$$

$$p_i^{\Gamma_{32}} = \frac{1}{2} p_i = -p_i^{\Gamma_{31}} \qquad \text{i=n, s} \tag{23}$$

where M_n, M_{ns}, Q_n and $\boldsymbol{p_i}$ refers to the beam axis while the tractions with superscripts Γ_{ij} are related to the local coordinate system defined on interface Γ_{ij}.

As the integrals are still performed on the interfaces and the collocation points are adopted on the beam axis, there is no problem of singularities.

4 Algebraic equations

The integral representations (11), (14) or (16) have to be transformed into algebraic expressions after discretizing the boundary and interfaces into elements. It has been adopted linear elements to approximate the problem geometry while the variables were approximated by quadratic shape functions.

Let us initially consider the simple bending analysis. Six values (w, ϕ_n, ϕ_s, Q_n, M_n and M_{ns}) are defined along the external boundary without beams, being three of them prescribed. Thus, in this case one has adopted to write eqn. (11) for an external collocation point very near to the boundary node. On the beam axis one has nine values: w, ϕ_n, ϕ_s, $\phi_{s,n}$, $\phi_{n,n}$, $w_{,n}$, Q_n, M_n and M_{ns} with collocation points adopted on the skeleton line coincident to the node or defined at element internal points when variable discontinuity is required at the element end. For external beams nodes the displacements $\phi_{s,n}$, $\phi_{n,n}$ and $w_{,n}$ are problem unknowns while three of the remaining values must be prescribed, requiring therefore six equations. In this case, one writes eqn. (11) plus the equations of $\phi_{s,n}$, $\phi_{n,n}$ and $w_{,n}$. for each collocation point. As all the nine values remain as unknowns for the internal beams nodes, in this case besides the equations adopted for the external beam nodes we also write the representations of Q_n, M_n and M_{ns}.

For the coupled stretching-bending problem, in addition to the values and equations defined previously for the simple bending problem one has to consider those related to the stretching problem. Along the external boundary without beams one has also the values u_s, u_n, p_n and p_s, being two of them prescribed. Thus in this case one has chosen to write eqns. (14) and (16) for each external collocation point. On beam nodes are also defined the following values: u_s, u_n, $u_{s,n}$, $u_{n,n}$, p_n and p_s All these values remain as unknowns in the internal beams, requiring therefore fifteen algebraic equations for each skeleton line point. In this case the adopted equations were those corresponding to the unknowns. For external beams, the displacements $u_{s,n}$ and $u_{n,n}$ are also problem unknowns while two of the four values: u_s, u_n, p_n and p_s, must be prescribed, leading to ten unknowns for each external beam node. It has been adopted to write eqns (14), (16) plus the following ones: $u_{s,n}$, $u_{n,n}$, $\phi_{s,n}$, $\phi_{n,n}$ and $w_{,n}$.

After writing the recommended algebraic relations one obtains the set of equations, which can be solved after applying the boundary conditions. For simple bending analysis and the coupled problem they are given, respectively by:

$$\underset{\sim}{H}^B \underset{\sim}{U}_B = \underset{\sim}{G}^B \underset{\sim}{P}_B + \underset{\sim}{T}^B \tag{24}$$

$$\begin{bmatrix} [H]^B & [\overline{H}]^S \\ --- & --- \\ [\overline{H}]^B & [H]^S \end{bmatrix} \begin{Bmatrix} \{U\}_B \\ \{U\}_S \end{Bmatrix} = \begin{bmatrix} [G]^B & [\overline{G}]^S \\ --- & --- \\ [0] & [G]^S \end{bmatrix} \begin{Bmatrix} \{P\}_B \\ \{P\}_S \end{Bmatrix} + \begin{Bmatrix} \{T\}^B \\ --- \\ \{T\}^S \end{Bmatrix} \tag{25}$$

where {U} and {P} are displacement and traction vectors, respectively; {T} is the independent vector due to the applied loads; [H] and [G] are matrices obtained by integrating all boundary and interfaces; B and S are related to bending and stretching problems.

In eqn. (25) the upper e bottom parts indicate, respectively, algebraic equations of the bending and stretching problems.

5 Numerical application

Let us consider the plate reinforced by two external beams depicted in figure 1, adopting $t_1=t_3=25cm$, $t_2=10.0cm$, Young's modulus $E=2.7x10^4 kN/cm^2$ and Poisson's ratio $\nu=0.0$. The two sides containing the beams are assumed free, while the other two are simply supported. The plate sides without beams as well as the beam axis were discretized by *12* quadratic elements (Figure 3), giving the total amount of 48 elements and 100 nodes. Observe that the element coincident to the beam width is automatically generated by the code.

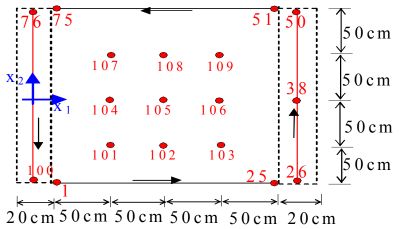

Figure 3: Discretization.

Table 1: Displacements at internal and boundary nodes.

Nodes	x_2 (cm)	w(cm) SB	w (cm) CP	ϕ_2 SB	ϕ_2 CP
1 to 25	-100	0	0.	-0.006667	0.007407
101, 102, 103, 94, 32	-50	0.25	-0.2777	-0.003333	0.003333
104, 105, 106, 38, 88	0	0.33	-0.3703	0.0	0.0

For both analyses one has prescribed appropriate boundary values to enforce constant curvatures over the entire structural element. So that displacements and internal forces would have exact solutions. For the simple bending analysis (see figure 1(c)) we have applied M_n=150 kNcm/cm and M_n= $2.34375x10^3$ kNcm/cm, respectively, along the simply supported plate boundary and on the beam simply supported ends (the beam width). The prescribed loads, for the coupled stretching-bending problem (see figure 1(b)), were: M_n=-$1.666667x10^5$Ncm/cm on the simply supported plate boundary; p_n=$3.75x10^5$kN/cm and M_n=-$5.416667x10^6$Ncm/cm along the beam width. As expected, for both analyses the computed values are exactly the theoretical ones (see table 1, where SB and CP mean, respectively, simple bending and coupled problem).

6 Conclusions

BEM formulations based on Reissner's hypothesis for analysing plate reinforced by beams have been presented. Some approximations for displacements and tractions along the beam cross section have been considered, leading to a model where the problem values are defined on the beam axis. The performance of the proposed formulation has been confirmed by comparing the results with analytical solutions.

Acknowledgements

The authors wish to thank FAPESP (São Paulo State Foundation for Scientific Research) for the financial support.

References

[1] Hu, C. & Hartley, G.A., Elastic analysis of thin plates with beam supports. *Engineering Analysis with Boundary Elements*, **13**, pp 229-238, 1994.

[2] Tanaka, M. & Bercin, A.N., A boundary Element Method applied to the elastic bending problems of stiffened plates. In: *Boundary Element Method XIX*, Eds. C.A. Brebbia et al., CMP, Southampton, 1997.

[3] Sapountzakis, E.J. & Katsikadelis, J.T., Analysis of plates reinforced with beams. *Computational Mechanics*, 26, pp 66-74, 2000.

[4] Fernandes, G.R & Venturini, W.S., Stiffened plate bending analysis by the boundary element method. *Computational Mechanics*, 28, pp 275-281, 2002.

[5] Fernandes, G.R. & Venturini, W. S., *Building floor analysis by the Boundary element method*. Computational Mechanics, 35, pp 277-291, 2005.

[6] Hartmann, F. & Zotemantel, R., The direct boundary element method in plate bending. *International Journal for Numerical Methods in Engineering*, **23(**11), pp 2049-2069, 1986

[7] Kirchhoff, G., Uber das gleichgewicht und die bewegung einer elastischen scleibe. *J. Math.,* 40, pp 51-58, 1850.

[8] Reissner, E., On bending of elastic plates. *Quart. Appl. Math*.; 5(1), pp 55-68; 1947

[9] Weën, F. V., Application of boundary integral equation method to Reissner's plate model. Int. J. Num. Meth. Emg., 18 (1), pp. 1-10, 1982.

[10] Palermo Jr. L., Plate bending analysis using the classical or the Reissner-Mindlin models. *Engineering Analysis with Boundary Elements*, **27**, pp. 603-609, 2003.