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FEM/BEM coupling for fluid-structure interaction including nonlinear effects

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Abstract

Various numerical procedures, with each respective merits and drawbacks, are available for the investigation of fluid-structure interaction problems. This paper concentrates on the coupling of the finite element method modeling the structure, and the boundary element method representing the fluid. Both methods are formulated in the time domain and a special algorithm is developed to realize the coupling. In particular, nonlinear effects, such as material nonlinearities or large displacements, may be taken into account in the structural model, while an infinite expansion of the fluid region is included in the boundary element formulation. Numerical examples exhibit the applicability of the new approach. It is observed that nonlinearities may have a considerable effect on the behavior of the fluid-structure system.

1 Introduction

Numerical analyses of the dynamical behavior of coupled fluid-structure systems demand the discretization of two subsystems: the structure and the fluid. Special attention must be paid when choosing a proper numerical method. The finite element method (FEM) is well applicable for considering complex structures, inhomogeneities, anisotropy of the material or other nonlinearities [1]. An infinite domain, however, cannot be modeled satisfactorily, since waves are reflected at the edges of the discretization if no special infinite finite elements are used [2, 3, 4]. The boundary element method (BEM), though is only useful for the simulation of linear material behavior, but is certainly well suited for modeling a semi-infinite domain

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since the influence of the energy radiation to infinity is included in the fundamental solution [5, 6, 7]. By combining these two methods — FEM for the structure and BEM for the unbounded fluid domain — the advantages of both procedures are exploited [8]. Direct calculation in the time domain enables not only the use of dynamical, non-harmonic excitations [9, 10, 11] but also the investigation of systems which include nonlinearities, such as a nonlinear material behavior, large deformations or unilateral boundary conditions [12, 13].

In the following, the finite element and the boundary element method are briefly introduced and formulated in a way that they may be used for the new coupling procedure. The general strategy is presented and an algorithm for the solution of the coupled system of equations is developed. The applicability of the procedure is illustrated with two numerical examples.

2 Finite element formulation

After discretization of the structural domain using finite elements, the equation of motion may be written as a matrix equation of the nodal displacements u and their respective accelerations $\ddot{u}$

$$M_F\ddot{u} + K_F(u)u = R_F, \tag{1}$$

where M_F is the mass matrix, $K_F(u)$ is the stiffness matrix, which might be displacement-dependent, and R_F denotes the vector of external forces. The solution for u and $\ddot{u}$ is found using the equilibrium of the effective external forces R_{eff} and the forces corresponding to the element stresses F:

$$R_{\text{eff}} - F = 0, \tag{2}$$

where $R_{\text{eff}} = R_F - M_F\ddot{u}$. For small displacements and a linear (Hookean) material law this equation may be solved directly. However, in the case of nonlinear effects, such as large displacements or nonlinear material laws, iterative solution schemes need to be employed. Details may be found, e. g., in [1].

3 Boundary element formulation

Assuming small displacements and neglecting volume forces, the scalar wave equation for a compressible, inviscid fluid in the absence of sources (acoustic

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medium) is given by

$$\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}. \tag{3}$$

Using weighted residuals in time and space, as shown by von Estorff and Antes [11], eqn (3) can be transformed into the integral representation

$$\begin{aligned} d(\xi)p(\xi;t) \quad = \quad & \int_0^t \oint_\Gamma (p^*(x,\xi;t,\tau)q(x;\tau) \\ & - q^*(x,\xi;t,\tau)p(x;\tau))\, d\Gamma\, d\tau, \end{aligned} \tag{4}$$

where p is the pressure, t the time and c the wave velocity. The integration has to be performed over the whole boundary Γ and the time t where $q = \frac{\partial p}{\partial n}$ is the flux, p^* and q^* are the corresponding fundamental solutions and d is a coefficient which depends on the boundary geometry. Discretizing space and time, this integral equation may be written as a matrix equation in p and q for each time step m when the source point ξ successively takes the location of each node. One obtains

$$\begin{aligned} \hat{H}^{(1)}p^{(m)} \quad = \quad & G^{(1)}q^{(m)} \\ & - \sum_{k=1}^{m-1}\left(H^{(m-k+1)}p^{(k)} - G^{(m-k+1)}q^{(k)}\right), \end{aligned} \tag{5}$$

where $G^{(j)}$, $H^{(j)}$ and $\hat{H}^{(j)}$ denote the so-called influence matrices calculated at time step j. Using $q = -\varrho\ddot{u}$, where ϱ is the fluid density, and introducing a constant matrix A transforming the fluid pressure p to nodal forces, the following equation is obtained:

$$\begin{aligned} \underbrace{Ap^{(m)}}_{F^{(m)}} = & \underbrace{-\varrho A[\hat{H}^{(1)}]^{-1}G^{(1)}}_{M_B}\,\ddot{u}^{(m)} \\ & - \sum_{k=1}^{m-1}\underbrace{\left(A[\hat{H}^{(1)}]^{-1}H^{(m-k+1)}p^{(k)} + \varrho A[\hat{H}^{(1)}]^{-1}G^{(m-k+1)}\ddot{u}^{(k)}\right)}_{R_k^{(m)}}. \end{aligned} \tag{6}$$

The factor M_B proceeding the node accelerations $\ddot{u}^{(m)}$ may be interpreted as the mass matrix of the boundary element subregion.

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4 Coupling strategy

For the analysis of a coupled system, the governing equations describing the structure and the fluid domain have to be solved simultaneously. Using the coupling conditions at each node of the interface (i)

$$\begin{aligned} R^F_{i,n} &= -Ap^B_i, \\ R^F_{i,t} &= 0, \\ \ddot{u}^F_{i,n} &= -\ddot{u}^B_i \end{aligned} \tag{7}$$

in normal (n) and tangential (t) directions the FE eqn (1) and the BE eqn (6) may be combined. R^F_i denotes the FE nodal forces, Ap^B_i the transformed BE pressures and $\ddot{u}^F_i$ and $\ddot{u}^B_i$ the accelerations of the FE and BE nodes, respectively. The different normal directions of the two subdomains are represented by negative signs.

Using these relations, the coupled system of equations may be written as

$$\underbrace{\begin{bmatrix} M_{\mathrm{F}} & 0 \\ 0 & M_{\mathrm{B}} \end{bmatrix}}_{M_{\mathrm{BF}}} \underbrace{\begin{bmatrix} \ddot{u}_{\mathrm{F}} \\ \ddot{u}_{\mathrm{BF}} \\ \ddot{u}_{\mathrm{B}} \end{bmatrix}}_{\ddot{u}} + \underbrace{\begin{bmatrix} K_{\mathrm{F}} & 0 \\ 0 & 0 \end{bmatrix}}_{K_{\mathrm{BF}}} \underbrace{\begin{bmatrix} u_{\mathrm{F}} \\ u_{\mathrm{BF}} \\ u_{\mathrm{B}} \end{bmatrix}}_{u} = \underbrace{\begin{bmatrix} R_{\mathrm{F}} \\ R_{\mathrm{BF}} \\ R_{\mathrm{B}} \end{bmatrix}}_{R}, \tag{8}$$

where the vectors and matrices are sorted with respect to the values of the uncoupled FE nodes (subscript F), the interface nodes (subscript BF) and the uncoupled BE nodes (subscript B). Note, that the total stiffness only contains the FE stiffness matrix. The two subsystems are coupled by overlapping the two mass matrices.

If $F_{\mathrm{F},n}$ denotes the force vector which corresponds to the FE element stresses at the iteration step n, the solution of the coupled system of equations for each time step may be found iteratively as described by the algorithm shown in Figure 1.

A sufficiently accurate solution is found if the norm of the unbalanced force vector $R_{\mathrm{eff}} - F_{\mathrm{F},n}$ is smaller than a given force tolerance *tol.* If the norm is too large, the solution must be improved with a further iteration step as described. It should be noted that the total effective stiffness matrix $K_{\mathrm{BF,eff}}$

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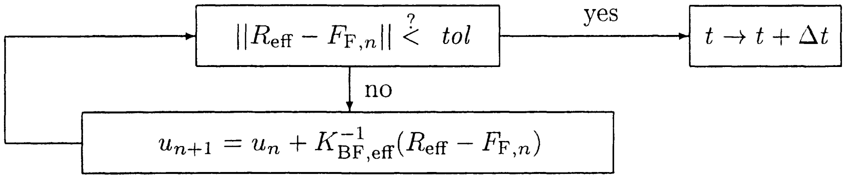
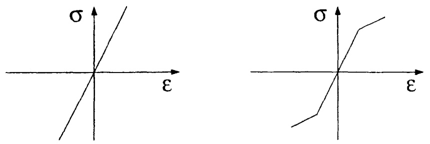


Figure 1: Algorithm for calculation of node displacements u.



$$W = \tfrac{1}{2}\kappa(J-1)^2 + \tfrac{1}{2}\mu(tr(C) - 3) - \mu \ln J$$

1) linear 2) piecewise linear 3) Neo-Hooke, compressible

Figure 2: Material laws assumed in the following numerical examples.

as well as the effective force R_{eff} depend on the coupled mass matrix M_{BF}:

$$K_{\text{BF,eff}} = K_{\text{BF}} + \frac{1}{\alpha \Delta t^2} M_{\text{BF}}, \qquad R_{\text{eff}} = R - M_{\text{BF}} \ddot{u}, \tag{9}$$

where α is the Newmark coefficient and Δt the size of the time step.

5 Numerical examples

The applicability of this procedure is shown by the following two numerical examples. In both cases nonlinearities in the FE domain and a semi-infinite BE domain are considered. For each case four different configurations are computed:

In the first configuration the structural behavior is determined while the influence of the fluid is neglected completely. For the remaining three cases, the coupled system is evaluated using different material laws for the whole FE domain, as shown in Figure 2.

The first material obeys Hooke's law, i. e., a linear relation between σ and ε is assumed. In the second case, the σ, ε-curve is nonlinear but piecewise linear while in the third case a compressible hyperelastic material (Neo-Hooke, [14]) is assumed. In the latter W denotes the strain-energy function, κ the bulk modulus, J the determinant of the Jacobian matrix, μ the shear modulus, C the right Cauchy-Green tensor, and *tr* the trace operator. Due to the assumption that the geometry may not change in longitudinal direction

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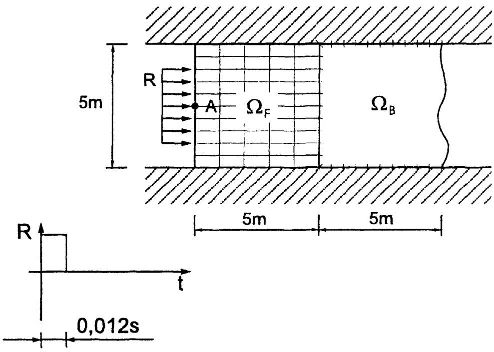

Structure (50 FE):
$E_0 = 2.66 \cdot 10^5$ kN/m^2
$\nu = 0.33$
$\varrho = 2.0$ t/m^3

Fluid (30 BE):
$\varrho = 1.0$ t/m^3
$c = 1436$ m/s

$\Delta t = 0.4$ ms

Figure 3: Quadratic body adjacent to a semi-infinite fluid domain.

(i. e., perpendicular to the depicted plane), the systems of the two examples can be described employing a plain strain formulation.

5.1 Quadratic body adjacent to a semi-infinite fluid domain

In the first example, a rectangular elastic region, discretized using 50 isoparametric linear finite elements, is clamped at its horizontal boundaries as shown in Figure 3.

Along the right-hand side, the elastic domain is coupled with a compressible fluid region of infinite expansion, which is represented by 30 linear boundary elements. The elastic domain is subjected to a distributed horizontal load of 1 kN/m, acting in time as a rectangular impulse during the first 30 time steps of $\Delta t = 0.4$ ms.

The transient behavior of the horizontal displacement at point A is shown in Figure 4.

Obviously the influence of the fluid on the displacement of the dam must not be neglected. Additionally, assuming linear material behavior for the structure in the coupled calculation (second curve), the FE domain is not described sufficiently accurate either. The dynamic behavior of the calculated displacement differs both qualitatively and quantitatively if one of the two nonlinear material laws is used.

5.2 Dam subject to earthquake excitation

The design of water retaining walls or dams often requires dynamic calculations. This has to be done to ensure that the structure will not fail even

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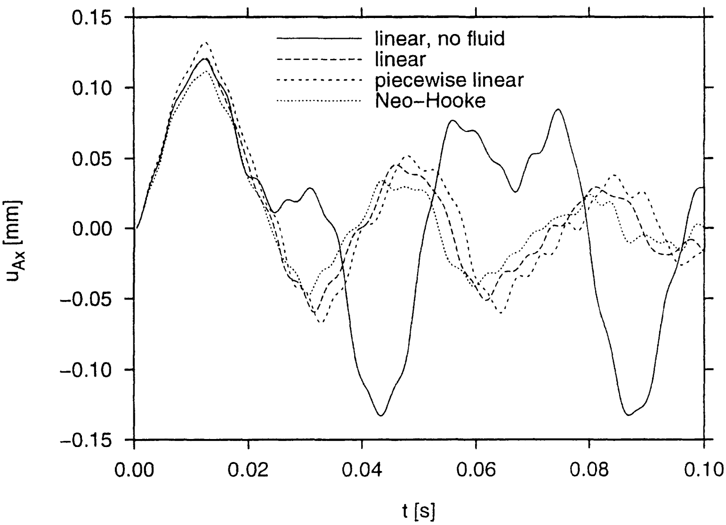
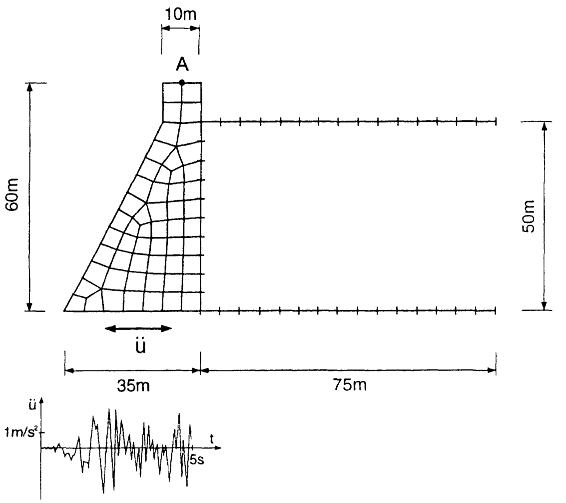


Figure 4: Horizontal displacement at point A due to an impulsive load.



Dam (54 FE):

$E_0 = 3.44 \cdot 10^6$ kN/m^2
$\nu = 0.25$
$\varrho = 2.0$ t/m^3

Storage-lake (40 BE):

$\varrho = 1.0$ t/m^3
$c = 1436$ m/s

$\Delta t = 0.02$ s

Figure 5: Dam with storage-lake.

in the case of an earthquake. To show how the new procedure can be used for these kind of problems, the dam-reservoir system given in Figure 5 is analyzed.

The considered dam is discretized with 54 isoparametric linear finite elements while the adjacent semi-infinite water reservoir is modeled with 40

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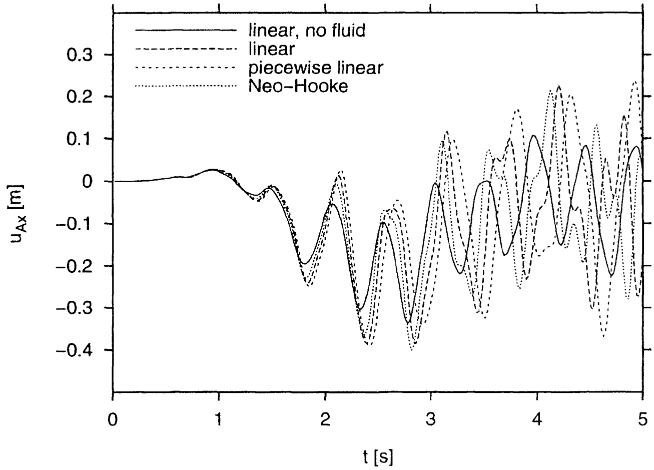

Figure 6: Displacement of the dam crest due to earthquake excitation.

linear boundary elements. The bottom of the dam is excited using the horizontal acceleration data measured during the El Centro earthquake. The horizontal displacement of the dam crest at point A is shown in Figure 6.

Also for this example it may be observed that the adjacent fluid is required to be taken into account. Moreover, nonlinear material characteristics assumed for the dam have a considerable impact on the solution.

6 Conclusion

A general time domain procedure has been presented to investigate transient fluid-structure interaction problems. The approach is able to deal with material nonlinearities and large deformations occurring in the structure. A semi-infinite extension of the adjacent fluid may possibly be simulated by a boundary element discretization.

The derived methodology has been applied to two numerical examples, namely a rectangular body linked to a semi-infinite fluid, and a water storage dam subject to earthquake excitation. Assuming different material laws, it could be shown that the new algorithm is highly flexible and gives reliable results. It is observed that structural nonlinearities may have considerable impact on the behavior of coupled fluid-structure systems.

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