This paper is part of the Proceedings of the 11th International Conference on Engineering Sciences (AFM 2016)
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Shoaling of long water waves by its interaction with a submerged breakwater of wavy surfaces

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Abstract

In this work, we analyze the shoaling of linear long water waves generated by a submerged breakwater composed of wavy surfaces. The undulating surfaces are described by sinusoidal profiles. The mathematical model is expressed in its dimensionless version. The effects of two different geometric parameters – the amplitude of the wavy surfaces and the submerged length of the breakwater – on the free surface elevation is analyzed. The wavy surfaces of the breakwater generate larger values of the wave amplitude than those obtained for breakwaters with flat surfaces. The asymptotic solution is compared with a numerical solution and the results are in good agreement. The results are presented for breakwaters with number of undulations of $m = n(= 1, 3)$. The present theory provides basis for comparison with other approximate theories based on shallow water flows and serves as a prelude to characterize submerged breakwaters of undulating surfaces.
Keywords: submerged breakwater, shallow flow, shoaling.

1 Introduction

For floating breakwaters, the geometry that is most commonly used is that of a rectangular prism. In this context, several relevant analytical, numerical and experimental investigations have been performed. Some pioneering studies of the interaction between water waves and rigid obstacles were conducted by [1–3]. Floating breakwaters have been constructed with several different geometric configurations, such as that studied by Kanoria [4], who addressed the scattering of surface water waves by a thick submerged rectangular wall with a gap in water of a finite depth. Floating obstacles are also used for a different purpose as provisional bridges, in which case they are known as pontoons. In this context, based on linear water waves, Drimer *et al.* [5] proposed a simplified two-dimensional analytical

www.witpress.com, ISSN 1743-3533 (on-line)
doi:10.2495/AFM160041

model to study the problem of the interaction between a type of floating breakwater and linear water waves; later, Abul-Azm and Gesraha [6] theoretically examined the hydrodynamic properties of a long rigid floating pontoon interacting with linear oblique waves in water of a finite arbitrary depth.

In this work, we obtain an asymptotic formula for the free water surface elevation of one-dimensional linear long waves interacting with a wide rectangular breakwater of wavy surfaces. The undulating surfaces are described by sinusoidal profiles. To obtain an asymptotic solution to the governing equations in the presence of wavy surfaces, the domain perturbation method is implemented. Analytical results are presented to illustrate the effects of the wave characteristics and structure parameters on the amplitude of the water waves. This mathematical model is only valid for breakwaters with number of undulations of $m = n(= 1, 3)$. An analytical solution for the velocity potential is obtained by applying a regular perturbation technique in combination with patching boundary conditions.

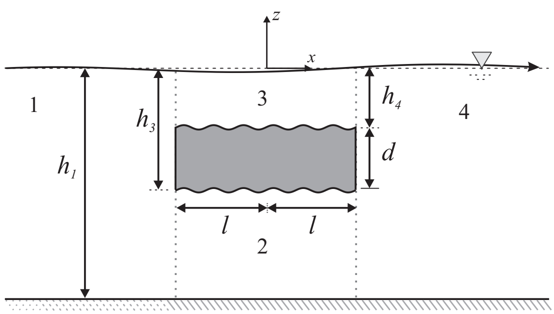

Figure 1: Side view of the physical model under study. The water wave amplitudes A_w, where $w = I$, R and T, represent the amplitudes of the incident (travelling from left to right), reflected and transmitted water waves, respectively.

2 Formulation

Let us consider the interaction between a one-dimensional linear long water wave propagating from left to right and a fixed, submerged, wide rectangular breakwater with wavy surfaces. Figure 1 presents the geometric configuration to be analyzed. In the selected Cartesian coordinate system, the positive direction of the x axis points to the right, with the origin in the middle of the structure. The z axis points outwards in the direction normal to the mean water level. The floating breakwater has a width of $2l$ and a length L_y normal to the $x - z$ plane, with $L_y \gg 2l$ and $L_y \gg \lambda$, where λ is the incident wavelength. The physical model consists of four regions, labelled 1, 2, 3 and 4; the constant depth in regions 1 and 4 is h_1, as can be appreciated from Fig. 1. The interval in which the structure serves to define regions 2 and 3 is $-l \leq x \leq l$, whereas regions 1 and 4 are defined by the intervals $-\infty < x \leq -l$ and $l \leq x < \infty$. In the physical domain, the fluid is considered to be incompressible, inviscid and irrotational. For an irrotational flow, the velocity components can be expressed in terms of a scalar function of the form

$$\Phi_j(x,t) = \text{Re}[\psi_j(x)\text{e}^{-i\omega t}] \quad \text{and} \quad \Phi_2(x,z,t) = \text{Re}[\psi_2(x,z)\text{e}^{-i\omega t}] \tag{1}$$

where $\psi_j(x)$ are the velocity potential on any cross section in regions $j = 1,3,4$ and $\psi_2(x,z)$ is the velocity potential in region 2, $i = \sqrt{-1}$, ω is the wave frequency and Re indicates "the real part of" the expression in square brackets.

2.1 Dimensionless governing equations

In the present mathematical model we adopt the methodology and formulation proposed by Medina-Rodríguez *et al.* [7], who analyze the hydrodynamic forces and the reflection and transmission coefficients of linear long waves interacting with a submerged breakwater, composed of wavy surfaces in presence of a step. To reduce the number of combinations that can be formed based on the physical variables, we prefer to express the governing equations in their dimensionless forms. Therefore, we introduce the following dimensionless variables

$$\begin{gathered}\chi_1 = \chi_4 = kx, \quad \chi = \frac{x}{2l}, \; Z_1 = Z_4 = \frac{z}{h_1}, \; Z_2 = \frac{z}{S_1}, \; Z_3 = \frac{z}{h_4}, \\ \overline{D}_{up}(\chi) = \frac{D_{up}(x)}{h_4}, \; \overline{D}_{mid}(\chi) = \frac{D_{mid}(x)}{S_1}, \; \overline{D}_{low}(\chi) = \frac{D_{low}(x)}{S_1}, \\ \phi_2(\chi, Z_2) = -\frac{\psi_2(x,z)}{igA_I/\omega} \text{ and } \phi_j(\chi) = -\frac{\psi_j(x)}{igA_I/\omega} \text{ for } j = 1,3,4\end{gathered} \tag{2}$$

and therefore the geometric distributions of the upper and middle, which are identified by the subscripts up and mid, respectively, follow sinusoidal profiles of the forms

$$\overline{D}_{up}(\chi) = 1 - \varepsilon_1 \sin\left[m\pi\left(\chi + \frac{1}{2}\right)\right], \tag{3}$$

and

$$\overline{D}_{mid}(\chi) = H_{mid} - \varepsilon \sin\left[n\pi\left(\chi + \frac{1}{2}\right)\right] \tag{4}$$

where $k = 2\pi/\lambda$ is the wavenumber, $H_{mid} = h_3/S_1$, $H_{low} = h_2/S_1$, $\varepsilon_1 = A_u/h_4$ and $S_1 = h_1 - h_3$, the parameter $\varepsilon = A_l/S_1 \ll 1$ with A_u and A_l been the physical amplitudes of the undulations. To conduct an asymptotic analysis, we consider that the parameter $\varepsilon \ll 1$ and, in Eq. (3), the parameter $\varepsilon_1 \sim O(\varepsilon)$. The physical interpretation of the limit of $\varepsilon \ll 1$ is that in region 2, the amplitude A_l is very small compared with the gap S_1, and in region 3, the amplitude A_u is very small compared with the depth h_4. Upon introducing the dimensionless variables defined into the Laplace equation and the corresponding kinematic boundary conditions, we obtain that the governing equations valid in the

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interval $-\frac{1}{2} \leq \chi \leq \frac{1}{2}$ can be expressed in the following form

$$\Gamma^2 \frac{\partial^2 \phi_2}{\partial \chi^2} + \frac{\partial^2 \phi_2}{\partial Z_2^2} = 0 \quad \text{for } -\overline{D}_{low}(\chi) < Z_2 < -\overline{D}_{mid}(\chi)\,, \tag{5}$$

with the boundary conditions

$$\frac{\partial \phi_2}{\partial Z_2} + \Gamma^2 \frac{\partial \overline{D}_{mid}(\chi)}{\partial \chi} \frac{\partial \phi_2}{\partial \chi} = 0 \quad \text{at} \quad Z_2 = -\overline{D}_{mid}(\chi) \tag{6}$$

and

$$\frac{\partial \phi_2}{\partial Z_2} + \Gamma^2 \frac{\partial \overline{D}_{low}(\chi)}{\partial \chi} \frac{\partial \phi_2}{\partial \chi} = 0 \quad \text{at} \quad Z_2 = -\overline{D}_{low}(\chi)\,. \tag{7}$$

The dimensionless shallow flow governing equations for region 3 can be written as

$$\frac{d}{d\chi}\left[\overline{D}_{up}(\chi) \frac{d\phi_3(\chi)}{d\chi}\right] + \frac{4\mu^2}{\gamma}\phi_3(\chi) = 0\,, \tag{8}$$

where $\Gamma = S_1/2l$ and $\gamma = h_4/h_1$. The free surface elevation in region 3 is given by the following equation

$$\overline{\eta}_3(\chi) = \phi_3(\chi)\,. \tag{9}$$

For the linear long wave approximation, the velocity potentials in regions 1 and 4 may be written in the following dimensionless form [8]:

$$\phi_1(\chi_1) = e^{i(\chi_1+\mu)} + Re^{-i(\chi_1+\mu)} \text{ and } \phi_4(\chi_4) = Te^{i(\chi_4-\mu)} \tag{10}$$

respectively, where $\mu = kl = 2\pi l/\lambda$, which represents the physical ratio between the half-length of the structure, l, and the wavelength, λ.

The boundary conditions required to solve the system consisting of Eqs. (5)–(10) are defined by the continuity of the pressures and mass fluxes at the interfaces between the different regions. These boundary conditions, when expressed in their dimensionless forms, are given as follows: at $\chi = -1/2$,

$$\phi_1 = \phi_2, \quad \phi_1 = \phi_3\,, \tag{11}$$

$$\int_{-1}^{0} \frac{d\phi_1}{d\chi_1} dZ_1 = \frac{\Pi_1}{2\mu} \int_{-H_{low}}^{-H_{mid}} \frac{\partial \phi_2}{\partial \chi} dZ_2 + \frac{\gamma}{2\mu} \int_{-1}^{0} \frac{d\phi_3}{d\chi} dZ_3\,, \tag{12}$$

and at $\chi = 1/2$,

$$\phi_2 = \phi_4, \quad \phi_3 = \phi_4 \tag{13}$$

$$\int_{-1}^{0} \frac{d\phi_4}{d\chi_4} dZ_4 = \frac{\Pi_1}{2\mu} \int_{-H_{low}}^{-H_{mid}} \frac{\partial \phi_2}{\partial \chi} dZ_2 + \frac{\gamma}{2\mu} \int_{-1}^{0} \frac{d\phi_3}{d\chi} dZ_3\,, \tag{14}$$

where $\Pi_1 = S_1/h_1$.

3 Asymptotic solution for $\varepsilon \ll 1$

The velocity potentials ϕ_j, for $j = 1, 2, 3, 4$, and the unknown coefficients R and T in Eq. (10) can be expressed in the form of the following regular asymptotic expansions:

$$\phi_j = \phi_{j,0} + \varepsilon\phi_{j,1} + \varepsilon^2\phi_{j,2} + O(\varepsilon^3) \text{ for } j = 1, 2, 3, 4, \tag{15}$$

$$R = R_0 + \varepsilon R_1 + \varepsilon^2 R_2 + O(\varepsilon^3) \text{ and } T = T_0 + \varepsilon T_1 + \varepsilon^2 T_2 + O(\varepsilon^3)\,. \tag{16}$$

3.1 Asymptotic solution for region 2

To obtain an asymptotic solution to Eq. (5), the domain perturbation method is used. This method provides an approximate form in which this problem can be solved for $\varepsilon \ll 1$. The basic concept is to replace the exact boundary conditions, Eqs. (6) and (7), with approximate boundary conditions that are asymptotically equivalent for $\varepsilon \ll 1$ but are now applied at the coordinate surfaces defined by $Z_2 = -H_{mid}$ and $Z_2 = -H_{low}$, [7,9]. The method of domain perturbations leads to a regular expansion in the small parameter as follow

$$\phi_2 = E_0 + 2F_0\chi + \varepsilon\left(\frac{2\Gamma F_0}{n\pi}\frac{\cosh\tilde{\Phi}}{\sinh\left(\frac{n\pi}{2}\right)}\right) - \varepsilon^2\left(\frac{n\pi\Gamma^2 F_0\cosh\left(2\tilde{\Phi}\right)}{2\sinh^2\left(\frac{n\pi\Gamma}{2}\right)}\sin\overline{\Phi}\right), \tag{17}$$

where $\overline{\Phi}(\chi) = 2n\pi\left(\chi + \frac{1}{2}\right)$ and $\tilde{\Phi}(\chi) = n\pi\Gamma\left(Z_2 + \frac{H_{low}+H_{mid}}{2}\right)$.

3.2 Asymptotic solution for region 3

Substituting the asymptotic expansion given in Eq. (15), for $j = 3$, into the linear long wave equation given in Eq. (8) leads to the following:
for $O(\varepsilon^0)$,

$$\frac{d^2\phi_{3,0}}{d\chi^2} + \frac{(2\mu)^2}{\gamma}\phi_{3,0} = 0\,; \tag{18}$$

for $O(\varepsilon^1)$,

$$\frac{d^2\phi_{3,1}}{d\chi^2} + \frac{(2\mu)^2}{\gamma}\phi_{3,1} = \frac{d}{d\chi}\left\{\sin\left[\Phi\left(\chi\right)\right]\frac{d\phi_{3,0}}{d\chi}\right\}\,; \tag{19}$$

and for $O(\varepsilon^2)$,

$$\frac{d^2\phi_{3,2}}{d\chi^2} + \frac{(2\mu)^2}{\gamma}\phi_{3,2} = \frac{d}{d\chi}\left\{\sin\left[\Phi\left(\chi\right)\right]\frac{d\phi_{3,1}}{\partial\chi}\right\}\,, \tag{20}$$

where $\Phi(\chi) = m\pi\left(\chi + \frac{1}{2}\right)$.

The analytical solutions to Eqs. (18)–(20) are trivial; and therefore, the velocity potential ϕ_3 is given by

$$\phi_3 = B_0 \cos(2\alpha\chi) + C_0 \sin(2\alpha\chi) + \varepsilon \begin{bmatrix} B_1 \cos(2\alpha\chi) + C_1 \sin(2\alpha\chi) + \\ B_0 \left[a_1 \sin\Phi(\chi)\cos(2\alpha\chi) + a_2 \cos\Phi(\chi)\sin(2\alpha\chi)\right] + \\ C_0 \left[a_1 \sin\Phi(\chi)\sin(2\alpha\chi) - a_2 \cos\Phi(\chi)\cos(2\alpha\chi)\right] \end{bmatrix} + \varepsilon^2 \left[B_2 \cos(2\alpha\chi) + C_2 \sin(2\alpha\chi) + G(\chi)\right], \qquad (21)$$

where

$$\alpha = \frac{\mu}{\sqrt{\gamma}}, \quad a_1 = \frac{1}{4\left[1 - \frac{(m\pi)^2}{16\alpha^2}\right]} \quad \text{and} \quad a_2 = \frac{\alpha}{m\pi}\left[\frac{1 - \frac{(m\pi)^2}{8\alpha^2}}{1 - \frac{(m\pi)^2}{16\alpha^2}}\right]. \qquad (22)$$

The variable $G(\chi)$ is given in appendix A; see Eq. (29).

3.3 Asymptotic solutions for regions 1 and 4

Upon substituting the expansions given by Eqs. (15) and (16) into Eq. (10), we find that the problem at $O(\varepsilon^0)$ for regions 1 and 4 is given by the following equations:

$$\phi_{1,0} = e^{i(\chi_1+\mu)} + R_0 e^{-i(\chi_1+\mu)} \quad \text{and} \quad \phi_{4,0} = T_0 e^{i(\chi_4-\mu)}\,; \qquad (23)$$

for $O(\varepsilon^1)$,

$$\phi_{1,1} = R_1 e^{-i(\chi_1+\mu)} \quad \text{and} \quad \phi_{4,1} = T_1 e^{i(\chi_4-\mu)} \qquad (24)$$

and for $O(\varepsilon^2)$,

$$\phi_{1,2} = R_2 e^{-i(\chi_1+\mu)} \quad \text{and} \quad \phi_{4,2} = T_2 e^{i(\chi_4-\mu)}\,. \qquad (25)$$

The constants R_0, T_0, R_1, T_1, R_2 and T_2 are obtained from a system of four equations, which is obtained by implementing the patching boundary conditions defined in Eqs. (11)–(14). Therefore, we obtain.

$$R_0 = \frac{i\left[\mu(\gamma-1)\cos\alpha + \Pi_1\sqrt{\gamma}\sin\alpha\right]\sin\alpha}{b_0} \text{ and } T_0 = \frac{\mu\sqrt{\gamma} + \Pi_1 \sin 2\alpha}{b_0}, \qquad (26)$$

where $b_0 = \left[\cos\alpha - i\sqrt{\gamma}\sin\alpha\right]\left[\mu\sqrt{\gamma}\cos\alpha + \{\Pi_1 - i\mu\}\sin\alpha\right]$. The constants E_0, F_0 B_0 and C_0 are given in appendix A; see Eqs. (30)–(31).

For the next-order solution, $O(\varepsilon^1)$, we have four unknown constants, R_1, B_1, C_1 and T_1; to obtain these, we use the facts that $\phi_1 = \phi_3$, from Eq. (11), and $\phi_3 = \phi_4$, from Eq. (13). In addition, we use Eqs. (12) and (14), which leads to a system of four equations. Solving this system of equations, we obtain that

$$R_1 = \frac{\theta_0 + i\theta_1}{b_1} \quad \text{and} \quad T_1 = \frac{\theta_2 + i\theta_3}{b_1} \,, \tag{27}$$

with

$$\begin{aligned}
\theta_0 &= \gamma\left(\delta_2 - \delta_1 \cos 2\alpha\right) + 2\mu\Omega_1\sqrt{\gamma}\sin 2\alpha \,,\\
\theta_1 &= \delta_1\sqrt{\gamma}\sin 2\alpha + 2\mu\left(\Omega_1\cos 2\alpha - \Omega_2\right) \,,\\
\theta_2 &= \gamma\left(\delta_2\cos 2\alpha - \delta_1\right) + 2\mu\Omega_2\sqrt{\gamma}\sin 2\alpha \,,\\
\theta_3 &= 2\mu\left(\Omega_2\cos 2\alpha - \Omega_1\right) - \delta_2\sqrt{\gamma}\sin 2\alpha \,,\\
b_1 &= 2(1+\gamma)\alpha\sin 2\alpha + i4\mu\cos 2\alpha \,,\\
\delta_1 &= \left(m\pi a_1 + 2a_2\alpha\right)\left(B_0\cos\alpha - C_0\sin\alpha\right) \,,\\
\delta_2 &= (-1)^m\left(m\pi a_1 + 2a_2\alpha\right)\left(B_0\cos\alpha + C_0\sin\alpha\right) \,,\\
\Omega_1 &= a_2\left(B_0\sin\alpha + C_0\cos\alpha\right)
\end{aligned}$$

and $\Omega_2 = (-1)^m a_2\left(B_0\sin\alpha - C_0\cos\alpha\right)$. The constants B_1 and C_1 are given in Appendix A; see Eq. (31). In a similar manner as before, for $O(\varepsilon^2)$, we have that the constants R_2 and T_2 are given by

$$R_2 = \frac{\theta_4 + i\theta_5}{b_2} \quad \text{and} \quad T_2 = \frac{\theta_6 + i\theta_7}{b_2} \,, \tag{28}$$

with

$$\begin{aligned}
\theta_4 &= \sqrt{\gamma}\left(\sigma_1\cos 2\alpha - \sigma_2\right) + q_1\gamma\sin 2\alpha \,,\\
\theta_5 &= \sqrt{\gamma}\left(q_1\cos 2\alpha - q_2\right) - \theta_1\sin 2\alpha \,,\\
\theta_6 &= \sqrt{\gamma}\left(\theta_1 - \sigma_2\cos 2\alpha\right) + q_2\gamma\sin 2\alpha \,,\\
\theta_7 &= \sigma_2\sin 2\alpha + \sqrt{\gamma}\left(q_2\cos 2\alpha - q_1\right) \,,\\
b_2 &= (1+\gamma)\sin 2\alpha + i2\sqrt{\gamma}\cos 2\alpha \,,\\
\sigma_1 &= \frac{n\pi\Gamma F_0\Pi_1}{\mu\tanh\left(\frac{n\pi\Gamma}{2}\right)} - \frac{\gamma G'\left(\chi = -1/2\right)}{2\mu}
\end{aligned}$$

and

$$\sigma_2 = \frac{n\pi\Gamma F_0\Pi_1}{\mu\tanh\left(\frac{n\pi\Gamma}{2}\right)} - \frac{\gamma G'(\chi=1/2)}{2\mu}.$$

The constants B_2 and C_2 are given in Appendix A; see Eq. (31).

4 Results

In this section, analytical results are presented to illustrate the effect of different wave characteristics and structural parameters on the free surface elevation $\overline{\eta}_3(\chi)$. We use the following typical parameters: the water depths are $h_1 = 5$ m, $h_3 = 4$ m and $h_4 = 1.5$ m; the maximum thickness is defined by $h_3 - h_4 = 2.5$ m and the maximum length of the structure is $2l = 30$ m; obtaining a ratio of $2l/h_1 = 6$, which is of the same order of magnitude to those used by [4, 10], and the wavelength is taken to be $\lambda = 100$ m; however, because of the present mathematical formulation is expressed in dimensionless form, many other representative values of the geometric parameters can be used.

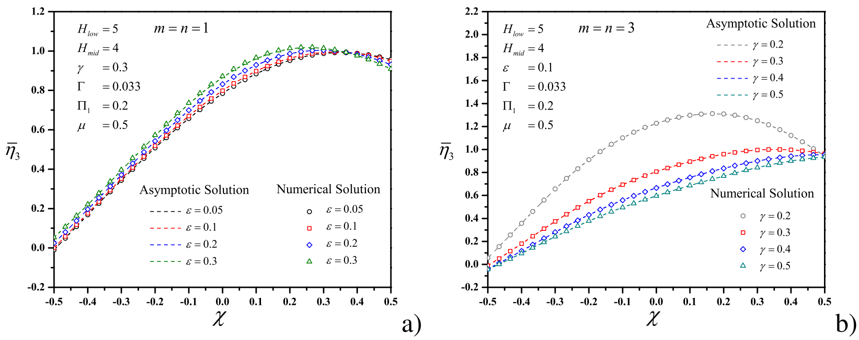

Figure 2: Free surface elevation $\overline{\eta}_3(\chi)$ for fixed values of $H_{low} = 5$, $H_{mid} = 4$, $\Gamma = 0.033$, $\Pi_1 = 0.2$, $\mu = 0.5$ for different values of a) $\varepsilon(= 0.05, 0.1, 0.2, 0.3)$ with $m = n = 1$ and b) $\gamma(= 0.2, 0.3, 0.4, 0.5)$ with $m = n = 3$.

In Fig. 2 is included the comparison of the asymptotic solution for the free surface elevation $\overline{\eta}_3(\chi)$ against the numerical solution of Eq. (8) as we see the results are in good agreement. We point out that the dependent variable in the ordinary differential equation (8) is the dimensionless velocity potential ϕ_3 and once it is obtained we use Eq. (9). We do not obtain the full numerical solution of Eq. (8); we only obtain, using the standard shooting numerical method, the numerical solution to Eq. (8) under the assumption that the boundary conditions are equal to the values obtained from the asymptotic solution and the integration interval is $\Delta\chi = 0.002$.

Figure 2a shows the variation of the dimensionless free surface elevation, which is calculated using Eq. (9), for different values of the parameter ε $(= 0.05, 0.1, 0.2, 0.3)$ and fixed values of the parameters $H_{low} = 5$, $H_{mid} = 4$, $\mu = 0.5$, $\gamma = 0.3$, $\Gamma = 0.033$, $\Pi_1 = 0.2$ and $m = n = 1$. In this figure can be observed that if the parameter ε increases, the free surface elevation $\overline{\eta}_3(\chi)$ slightly increases in the interval $0 \leq \chi < 0.36$ and decrease as $\chi \to 0.5$. For the three values of ε at $\chi = 0.36$ the height of the free surface elevation are equal. The amplification of the waves is because of the reduction of the depth in region 3, see Fig. 1. In Fig. 2b is shown the influence of the parameter $\gamma(= 0.2, 0.3, 0.4, 0.5)$, with $m = n = 3$, on the free surface elevation $\overline{\eta}_3(\chi)$, for the same fixed values used in Fig. 2a. In this figure clearly can be appreciated that for a breakwater located near to the water surface, $\gamma = 0.2$, the amplitude of the waves are larger than those obtained when the breakwater is near to the bottom, $\gamma = 0.5$; however, at $\chi \to 0.5$ the height of $\overline{\eta}_3(\chi)$ for the three values of the parameter γ are equal. The results shown that for small decrements of the parameter γ the height of the free surface elevation is increased significantly, as an example, at $\chi = 0.15$ and $\gamma(= 0.2, 0.05)$ the free surface elevation takes values of $\overline{\eta}_3(= 1.3, 0.6)$, respectively, the first value of $\overline{\eta}_3$ is 2.16 times greater than the second value. The present mathematical model is only valid for small values of the parameter ε, that is $\varepsilon \ll 1$, the case of $\varepsilon \sim O(1)$ is out of the scope of the present work.

5 Conclusions

In this work, we carried out an asymptotic analysis, at up to second order in the regular expansion, of the interaction of linear long waves with an impermeable, fixed, submerged breakwater composed of wavy surfaces. A formula for the free surface elevation is obtained. The validity of this mathematical formulation is restricted to the linear long wave approximation and for the number of undulations of the breakwater $m = n(= 1, 3)$. The present proposal of a submerged breakwater with wavy surfaces constitutes a new alternative structure for the shoaling of water waves.

Acknowledgement

This research is financially supported by project SIP: 20161477.

Appendix A

$$\begin{aligned} G(\chi) = {} & \cos\varphi \left(\nu_1(B_0 + C_0\varphi) + \nu_2 \cos 2\Phi + \nu_3 \sin 2\Phi\right) + \\ & \nu_4 \cos\left[\Phi - \varphi\right] + \nu_5 \sin\left[\Phi - \varphi\right] + \nu_6 \sin\varphi 2\Phi + \\ & \nu_7 \left(\nu_{10} \sin\left[2\Phi - \varphi\right] + \nu_{11} \cos 2\Phi \sin\varphi\right) \ , \end{aligned} \tag{29}$$

where

$$\begin{gathered} \nu_1 = 4m\pi\left(m^2\pi^2 - 16\alpha^2\right)(2\alpha\psi_1 + m\pi\psi_2), \\ \nu_2 = 8\alpha B_0\left(m^2\pi^2 - 16\alpha^2\right)(m\pi\psi_1 + \alpha\psi_2), \\ \nu_3 = 4\alpha C_0\left(16\alpha^2 - m^2\pi^2\right)(\alpha + m\pi)(\psi_1 + \psi_2), \\ \nu_4 = 32\alpha^2 C_1\left(8\alpha^2 - (m\pi)^2\right), \qquad \nu_5 = -128\alpha^3 B_1 m\pi, \\ \nu_6 = 4\alpha B_0\left(m^2\pi^2 - 16\alpha^2\right), \qquad \nu_7 = 2\alpha C_0\left(16\alpha - m^2\pi^2\right), \\ \nu_8 = 2m\pi(2\alpha\psi_1 + m\pi\psi_2), \qquad \nu_9 = 2(\alpha\psi_1 + m\pi\psi_2), \\ \nu_{10} = 2(\alpha - m\pi)(\psi_1 - \psi_2), \quad \text{and} \quad \nu_{11} = 2(\alpha + m\pi)(\psi_1 + \psi_2) \end{gathered}$$

$$B_0 = \frac{1}{\cos\alpha - i\sqrt{\gamma}\sin\alpha}, \ \ C_0 = -\frac{\mu}{\mu\sin\alpha + \left[\mu\sqrt{\gamma}\cos\alpha + \Pi_1\sin\alpha\right] i} \tag{30}$$

$$E_0 = B_0 \cos\alpha \text{ and } F_0 = C_0 \sin\alpha,\; B_1 = \frac{\gamma(\delta_2-\delta_1)-2i\mu(\Omega_1+\Omega_2)}{4\mu\left(i\cos\alpha+\sqrt{\gamma}\sin\alpha\right)},$$
$$C_1 = -\frac{\gamma(\delta_1+\delta_2)+2i\mu(\Omega_1-\Omega_2)}{4\mu\left(\sqrt{\gamma}\cos\alpha-4i\sin\alpha\right)},\; B_2 = -\frac{q_1+q_2+i(\sigma_1-\sigma_2)}{2\left(\cos 2\alpha-i\sqrt{\gamma}\sin 2\alpha\right)} \text{ and} \tag{31}$$
$$C_2 = \frac{\sigma_1+\sigma_2+i(q_2-q_1)}{2\left(\sqrt{\gamma}\cos 2\alpha-i\sin 2\alpha\right)},$$

where $\hat{\sigma}_1 = \tilde{a} - \frac{\gamma G'(\chi=-1/2)}{2\mu}$ and $\hat{\sigma}_2 = \tilde{a} - \frac{\gamma G'(\chi=1/2)}{2\mu}$ with $\tilde{a} = \frac{n\pi\Gamma F_0\Pi_1}{2\mu\tanh(n\pi\Gamma)}$.

$$\begin{aligned}
\Theta_0 &= \gamma\left(\delta_2 - \delta_1\cos 2\alpha\right) + 2\mu\Omega_1\sqrt{\gamma}\sin 2\alpha\\
\Theta_1 &= \delta_1\sqrt{\gamma}\sin 2\alpha + 2\mu\left(\Omega_1\cos 2\alpha - \Omega_2\right)\\
\Theta_2 &= \sqrt{\gamma}\left(\bar{\sigma}_1\cos 2\alpha - \bar{\sigma}_2\right) + q_1\gamma\sin 2\alpha\\
\Theta_3 &= \sqrt{\gamma}\left(q_1\cos 2\alpha - q_2\right) - \bar{\sigma}_1\sin 2\alpha\\
\Theta_4 &= \gamma\left(\delta_2\cos 2\alpha - \delta_1\right) + 2\mu\Omega_2\sqrt{\gamma}\sin 2\alpha\\
\Theta_5 &= 2\mu\left(\Omega_2\cos 2\alpha - \Omega_1\right) - \delta_2\sqrt{\gamma}\sin 2\alpha\\
\Theta_6 &= \sqrt{\gamma}\left(\bar{\sigma}_1 - \bar{\sigma}_2\cos 2\alpha\right) + q_2\gamma\sin 2\alpha\\
\Theta_7 &= \bar{\sigma}_2\sin 2\alpha + \sqrt{\gamma}\left(q_2\cos 2\alpha - q_1\right)\\
M_0 &= M_1 = 2\alpha(1+\gamma)\sin 2\alpha + 4i\mu\cos 2\alpha\\
M_1 &= M_3 = (1+\gamma)\sin 2\alpha + 2i\cos 2\alpha
\end{aligned} \tag{32}$$

where $\bar{\sigma}_1 = \tilde{b} - \frac{\gamma G'(\chi=-1/2)}{2\mu}$, $\bar{\sigma}_2 = \tilde{b} - \frac{\gamma G'(\chi=1/2)}{2\mu}$ and $\tilde{b} = \frac{n\pi\Gamma F_0}{2\mu H_l\tanh[n\pi\Gamma]}$.

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