Hydrodynamic loading on elliptic cylinders in regular waves

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Abstract

This paper deals with an elliptical cylinder in regular waves and investigation is made to determine the exciting forces and moments exerted on this body in regular waves. We have used the potential theory formulation in this case under the assumption that the viscous effects are negligible. An analytical solution to the linear wave diffraction problem, in terms of the infinite series of Mathieu's function, for a fixed vertical cylinder of elliptical cross-section in water of finite depth d has been presented. Further, Mathieu functions were simplified by taking the characteristic number s to be independent of the parameter q. The values of these simplified forms of Mathieu functions are substituted in the closed form approximations for the force and moment components for small eccentricity e of the cylinder, and the results thus obtained are compared with the existing results in the previous literature. The comparison shows good agreement except for $e = 0.9$ and $e = 1.0$. The limiting case of the circular cylinder, obtained by taking $e = 0$, has also been studied. The horizontal and vertical forces for the circular and elliptical cylinder for angle of incidence $\alpha = 0°$ and $90°$ have been compared.
Keywords: hydrodynamics, loading, forces and moments, elliptic cylinders, regular waves, finite depth oceans, diffraction, scattering.

1 Introduction

The determination of wave forces on offshore structures is essential to study the effects of waves, wind and current on them. The offshore structures should experience minimal movement to provide a stable work station for operations such as drilling and production of oil. The solution to the problem of ocean wave interaction with offshore structures is usually very complex. In many cases, only

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an approximate solution is sought. Some of the mathematical techniques required for the hydrodynamic problem associated with the design of offshore structures are analytical while many are numerical in nature. While the evolution of computers has made the numerical methods more advantageous over the classical analytical methods, numerical methods alone cannot find absolute success without being complemented by either analytical methods or at least experiments; in this sense analytical methods become a cost efficient and handy technique for designers in most cases.

The diffraction of plane waves by circular cylinders and ribbons is well known and adequately documented in the literature. Both situations are the limiting cases of the corresponding elliptical-cylinder problem. Many studies have been done on the interaction of electromagnetic or sound waves with an elliptic cylinder of infinite length. The solution to the problem of diffraction of electromagnetic waves by an elliptic cylinder and the corresponding Mathieu function series solution was originally given by Sieger [4]. This paper attracted scant attention, owing possibly to a lack of physical applications, and to analytical difficulties; for the Mathieu functions could not be treated in a straight forward way like Bessel functions or Legendre polynomials.

Chen and Mei [1] investigated the problem of scattering of linear progressive waves by an elliptic cylinder. They obtained the various force and moment coefficients in terms of Mathieu function series and presented extensive numerical results for arbitrary wavelengths using a computer program developed by Clemm [2] for the Mathieu functions.

In the present work the theory developed by Williams [5] has been used to obtain the force and moment coefficients on the surface of a fixed elliptic cylinder in water of finite depth for various angles of incidence and for a wide range of eccentricities ranging from 0.1 to 1.0. While deriving the approximate expressions for the force and moment coefficients for small eccentricity Williams [5] has adopted the notations of McLachlan [3] while in the present analysis simplified forms of Mathieu functions given.

2 The coordinate system

Consider a fixed, rigid vertical cylinder of elliptical cross-section in water of finite depth d (Figure 1). The coordinate system is fixed with the x-axis along the still water surface and the z-axis pointing vertically upwards along the axis of the cylinder. Since the objective of this investigation is to study the potential flow around an elliptical cylinder in the x-y plane, therefore, the elliptical coordinate system would be the appropriate and logical choice. The elliptical coordinates are denoted by (ξ, η, z) where ξ = constant and η = constant are families of confocal ellipses and hyperbolae, intersecting orthogonally, with common foci ($\pm c$, 0).

The elliptical coordinates (ξ, η, z) can be related to the Cartesian coordinates (x, y, z) by using the following transformation. $x = c\cosh\xi\cos\eta$, $y = c\sinh\xi\sin\eta$ and $z = z$. The semi-major and minor axes of the ellipse are given as follows. $a = c\cosh\xi$ and $b = c\sinh\xi$. The eccentricity of the ellipse can be expressed as

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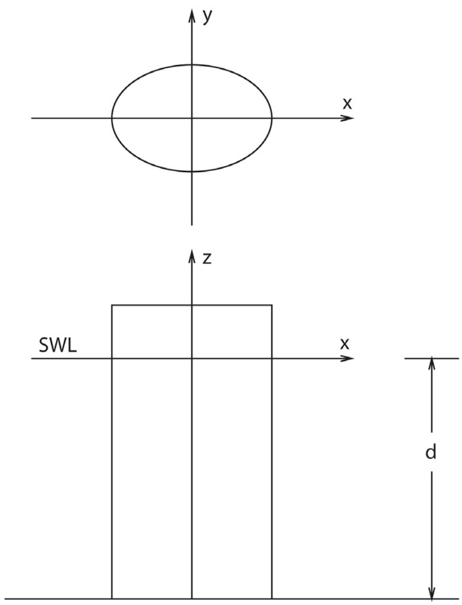

Figure 1: Definition sketch for an elliptic cylinder.

$e = \sqrt{1-(b/a)^2}$. The focal distance is given by $c = \sqrt{a^2 - b^2}$. If we keep focus, c, fixed and let eccentricity $e \to 1, \xi \to 0$, the elliptical cylinder transforms to a line of length $2c$. Alternatively, if we allow $e \to 0, \xi \to \infty$, then $a \to b$, the ellipse will transform to a circle with radius $r = (x^2 + y^2)^{\frac{1}{2}}$.

3 Mathematical formulation

A simple and concise mathematical model to study the potential flow around an elliptic cylinder can be constructed by using the well known general assumptions which govern any flow around a solid geometry. To facilitate this, we consider an inviscid, homogeneous and incompressible fluid and the flow around the object to be irrotational. By virtue of these assumptions, Euler's and the continuity equations can be simplified to yield Laplace's equation. Let $\Phi(\xi, \eta, z, t)$ denote the total fluid potential and let $z = \gamma(\xi, \eta, t)$ be the equation of the free surface, where (ξ, η, z) are the elliptical coordinates. Then everywhere in the region of the flow, the fluid motion is governed by Laplace's equation.

$$\nabla^2 \Phi(\xi, \eta, z, t) = 0 \tag{1}$$

where ∇ is the Laplacian operator.

The velocity potential is subject to the following linearized boundary conditions:

Bottom boundary condition:

Assuming the floor of the ocean to be flat, the boundary condition at the ocean bottom states that the vertical component of the velocity is zero at the bottom

$$\frac{\partial \Phi}{\partial z} = 0 \quad \text{at } z = -d; \tag{2}$$

Body surface boundary condition:

On the surface of the body the velocity of the fluid must be equal to zero

$$\frac{\partial \Phi}{\partial \xi} = 0 \quad \text{on } \xi = \xi_o \tag{3}$$

Dynamic free surface boundary condition:

The dynamic free surface condition is derived from the Bernoulli equation, on the assumption that the atmospheric pressure outside the fluid is constant.

$$g\gamma - \frac{\partial \Phi}{\partial t} = 0 \quad \text{on } z = 0 \tag{4}$$

Kinematic free surface boundary condition:

The kinematic condition states that a particle lying on the free surface will continue to remain on the surface. Mathematically,

$$\frac{\partial \gamma}{\partial t} + \frac{\partial \Phi}{\partial z} = 0 \quad \text{on } z = \gamma \tag{5}$$

Equation (5) is a concise form of kinematic free surface boundary condition from which the nonlinear terms have been omitted. The dynamic and the kinematic free surface boundary conditions can be combined together to yield one equation which is known as free surface boundary condition,

$$\frac{\partial^2 \Phi}{\partial t^2} + g\frac{\partial \Phi}{\partial z} = 0 \quad \text{on } z = 0 \tag{6}$$

Radiation condition:

In addition to the above boundary conditions, there is the Sommerfeld radiation condition to be discussed. This radiation condition is introduced after splitting the total velocity potential Φ as the sum of an incident wave potential Φ_I and the scattered wave potential Φ_S that is

$$\Phi = \Phi_I + \Phi_S \tag{7}$$

At large distance from the cylinder the scattered potential must be an outgoing wave i.e Φ_S must satisfy the radiation condition, namely

$$\frac{\partial \phi_S}{\partial r} \pm ik_1\phi_S = 0 \quad \text{as } r \to \pm\infty \tag{8}$$

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where $\Phi_s = Re(\phi_s e^{-i\sigma t)})$ and k_1 is the incident wave number. Here Re stands for the real part, σ is the wave frequency and ϕ_s the complex wave potential.

4 Solution of Laplace's equation in elliptical coordinates

Laplace's equation in cartesian coordinates (x, y, z) is written as:

$$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0 \tag{9}$$

The above equation can be expressed in terms of elliptical coordinates (ξ, η,z) by using the transformation stated in section 2:

$$\frac{\partial^2\Phi}{\partial \xi^2} + \frac{\partial^2\Phi}{\partial \eta^2} + c^2(\sinh^2\xi + \sin^2\eta)\frac{\partial^2\Phi}{\partial z^2} = 0 \tag{10}$$

To obtain a solution to equation (10) we write

$$\Phi(\xi, \eta, z) = F(\xi)G(\eta)Z(z) \tag{11}$$

where, F(ξ) is a function of ξ, G(η) is a function of η and Z(z) is a function of z alone. Defining a new positive constant s and the parameter q as $s = l^2 + \frac{p^2c^2}{2}$ and $q = \frac{p^2c^2}{4}$, the Laplace's equation can be written as

$$\frac{d^2G}{d\eta^2} + (s - 2q\cos 2\eta)G = 0 \tag{12}$$

and

$$\frac{d^2F}{d\xi^2} - (s - 2q\cosh 2\xi)F = 0 \tag{13}$$

Equation (12) and (13) are known as the canonical forms of Mathieu's equation and modified Mathieu's equation respectively. The solution to equation (12) consists of the periodic Mathieu functions

$$G(\eta) = [ce_n(\eta, q), se_n(\eta, q)] \tag{14}$$

where $ce_n(\eta, q)$ and $se_n(\eta, q)$ are respectively the even and odd Mathieu functions of order n and argument η, while the solution to equation (13) consists of the modified (or radial) Mathieu functions

$$F(\xi) = [Ce_n(\xi, q), Se_n(\xi, q); Fey_n(\xi, q), Gey_n(\xi, q)] \tag{15}$$

of the first and second kind of order n and arguments ξ. These and other variety of Mathieu functions are defined in McLachlan . The functions $ce_n(\eta, q)$ and $se_n(\eta, q)$ can be expressed as a series of cosine and sine terms. Also, the functions $Ce_n(\xi, q)$ and $Se_n(\xi, q)$ can be written as a series of Bessel's function J_n of the first kind, while the functions $Fey_n(\xi, q)$ and $Gey_n(\xi, q)$ can be written as a series

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of Bessel's function Y_n of the second kind. Thus solution of equation (10) which is periodic in η and has a real exponential behaviour in z will consist of the product of equation (11) with any linear combination of products of equations (14) and (15) which have the same value of s and q, namely

$$[\exp(pz), \exp(-pz)][(Ce_n(\xi,q), Fey_n(\xi,q))ce_n(\eta,q), \\ (Se_n(\xi,q), Gey_n(\xi,q))se_n(\eta,q)]$$

5 A close form analytical solution

In this section a closed form solution to the linear diffraction problem is obtained for a fixed vertical cylinder of elliptic cross-section extending from the seabed and piercing the free surface. This type of analysis, was initially done by Chen and Mei [1], and then by Williams [5]. A complete derivation of the velocity potential Φ and expressions for the force and moment coefficients is presented here, in terms of Mathieu functions. For the following analysis, the usual assumption of inviscid, incompressible fluid and irrotational flow is made. A linear simple harmonic wave of time period T, wave length L, wave height H and angular frequency σ is incident at an angle α to the major-axis upon a vertical cylinder of elliptic cross-section in water of finite depth d. The incident wave upon arriving at the structure undergoes significant diffraction. The incident waves are assumed to be of small height as compared to their lengths in a finite water depth so that the linear theory may be used. The coordinate system and the mathematical formulation along with the boundary conditions have already been discussed. Then, the potential due to the linear incident wave is given by

$$\Phi_I = \frac{gH}{\sigma}\frac{\cosh k_1(z+d)}{\cosh k_1 d} Re\left[e^{-i\sigma t}\exp\{ik_1(x\cos\alpha + y\sin\alpha)\}\right] \tag{16}$$

where Re stands for the real part, x and y are given in section 2 and g is the acceleration due to gravity. In terms of Mathieu functions (see McLachlan [3, p.207]),

$$\begin{aligned}\Phi_I = \frac{2gH}{\sigma}\frac{\cosh k_1(z+d)}{\cosh k_1 d} \\ Re\Bigg[e^{-i\sigma t}\sum_{n=0}^{\infty}\Bigg\{ A_0^{2n}\frac{Ce_{2n}(\xi,q)ce_{2n}(\eta,q)ce_{2n}(\alpha,q)}{ce_{2n}(0,q)ce_{2n}(\frac{\pi}{2},q)} \\ + qB_2^{2n+2}\frac{Se_{2n+2}(\xi,q)se_{2n+2}(\eta,q)se_{2n+2}(\alpha,q)}{se'_{2n+2}(0,q)se'_{2n+2}(\frac{\pi}{2},q)} \\ - iq^{\frac{1}{2}}A_1^{2n+1}\frac{Ce_{2n+1}(\xi,q)ce_{2n+1}(\eta,q)ce_{2n+1}(\alpha,q)}{ce_{2n+1}(0,q)ce'_{2n+1}(\frac{\pi}{2},q)} \\ + iq^{\frac{1}{2}}B_1^{2n+1}\frac{Se_{2n+1}(\xi,q)se_{2n+1}(\eta,q)se_{2n+1}(\alpha,q)}{se'_{2n+1}(0,q)se_{2n+1}(\frac{\pi}{2},q)}\Bigg\}\Bigg]\end{aligned} \tag{17}$$

In order to find an expression for the scattered potential Φ_S we introduce the modified Mathieu functions of the third and fourth kinds denoted by $Me_n^{(1)}$, $Ne_n^{(1)}$ and $Me_n^{(2)}$, $Ne_n^{(2)}$ which can be expressed as:

$$\begin{aligned} Me_n^{(1),(2)} &= Ce_n(z,q) \pm iFey_n(z,q) \\ Ne_n^{(1),(2)} &= Se_n(z,q) \pm iGey_n(z,q) \end{aligned} \tag{18}$$

where Fey_n and Gey_n are the even and odd modified Mathieu functions of the second kind respectively.

Thus, the scattered potential Φ_S is taken to be of the form

$$\begin{aligned} \Phi_S = \frac{2gH}{\sigma}\frac{\cosh k_1(z+d)}{\cosh k_1 d} \\ Re\Bigg[e^{-i\sigma t}\sum_{n=0}^{\infty}\Big\{c_{2n}Me_{2n}^{(1)}(\xi,q)ce_{2n}(\eta,q)ce_{2n}(\alpha,q) \\ + c_{2n+1}Me_{2n+1}^{(1)}(\xi,q)ce_{2n+1}(\eta,q)ce_{2n+1}(\alpha,q) \\ + s_{2n+2}Ne_{2n+2}^{(1)}(\xi,q)se_{2n+2}(\eta,q)se_{2n+2}(\alpha,q) \\ + s_{2n+1}Ne_{2n+1}^{(1)}(\xi,q)se_{2n+1}(\eta,q)se_{2n+1}(\alpha,q)\Big\}\Bigg] \end{aligned} \tag{19}$$

where c_{2n}, c_{2n+1}, s_{2n+2} and s_{2n+1} are coefficients to be determined. The total velocity potential is obtained by adding the incident potential Φ_I and scattered potential Φ_S

$$\Phi = \Phi_I + \Phi_S \tag{20}$$

Thus, using equation (17) and (19), we obtain:

$$\begin{aligned} \Phi = \frac{2gH}{\sigma}\frac{\cosh k_1(z+d)}{\cosh k_1 d} \\ Re\Bigg\{e^{-i\sigma t}\sum_{n=0}^{\infty}\Bigg[\left\{\frac{A_0^{2n}Ce_{2n}(\xi,q)}{ce_{2n}(0,q)ce_{2n}(\frac{\pi}{2},q)} + c_{2n}Me_{2n}^{(1)}(\xi,q)\right\} \\ \times ce_{2n}(\eta,q)ce_{2n}(\alpha,q) \\ + \left\{\frac{qB_2^{2n+2}Se_{2n+2}(\xi,q)}{se'_{2n+2}(0,q)se'_{2n+2}(\frac{\pi}{2},q)} + s_{2n+2}Ne_{2n+2}^{(1)}(\xi,q)\right\} \\ \times se_{2n+2}(\eta,q)se_{2n+2}(\alpha,q) \\ + \left\{\frac{-iq^{\frac{1}{2}}A_1^{2n+1}Ce_{2n+1}(\xi,q)}{ce_{2n+1}(0,q)ce'_{2n+1}(\frac{\pi}{2},q)} + c_{2n+1}Me_{2n+1}^{(1)}(\xi,q)\right\} \\ \times ce_{2n+1}(\eta,q)ce_{2n+1}(\alpha,q) \end{aligned}$$

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$$+\left\{\frac{iq^{\frac{1}{2}}B_1^{2n+1}Se_{2n+1}(\xi,q)}{se'_{2n+1}(0,q)se_{2n+1}(\frac{\pi}{2},q)}+s_{2n+1}Ne_{2n+1}^{(1)}(\xi,q)\right\}$$
$$\times\, se_{2n+1}(\eta,q)se_{2n+1}(\alpha,q)\Big]\Big]\Big\} \qquad (21)$$

To determine the coefficients, $c_{2n}, c_{2n+1}, s_{2n+2}$ and s_{2n+1} we use the boundary condition that the water-particle velocity normal to the surface of the cylinder is zero. Thus we obtain

$$\Phi=\frac{2gH}{\sigma}\frac{\cosh k_1(z+d)}{coshk_1d}$$
$$\sum_{n=0}^{\infty}\Bigg[\left\{\frac{A_0^{2n}ce_{2n}(\eta,q)ce_{2n}(\alpha,q)Ee_{2n}(\xi,q)Ve_{2n}(\xi_0,q)}{ce_{2n}(0,q)ce_{2n}(\frac{\pi}{2},q)}\right\}\cos(\sigma t-\beta_{2n})$$
$$+\left\{\frac{qB_2^{2n+2}se_{2n+2}(\eta,q)se_{2n+2}(\alpha,q)Ue_{2n+2}(\xi_0,q)De_{2n+2}(\xi,q)}{se'_{2n+2}(0,q)se'_{2n+2}(\frac{\pi}{2},q)}\right\}$$
$$\times\cos(\sigma t-\gamma_{2n+2})$$
$$+\left\{\frac{q^{\frac{1}{2}}A_1^{2n+1}ce_{2n+1}(\eta,q)ce_{2n+1}(\alpha,q)Ee_{2n+1}(\xi,q)Ve_{2n+1}(\xi_0,q)}{ce_{2n+1}(0,q)ce'_{2n+1}(\frac{\pi}{2},q)}\right\}$$
$$\times\sin(\beta_{2n+1}-\sigma t)$$
$$+\left\{\frac{q^{\frac{1}{2}}B_1^{2n+1}se_{2n+1}(\eta,q)se_{2n+1}(\alpha,q)Ue_{2n+1}(\xi_0,q)De_{2n+1}(\xi,q)}{se'_{2n+1}(0,q)se_{2n+1}(\frac{\pi}{2},q)}\right\}$$
$$\times\sin(\sigma t-\gamma_{2n+1})\Bigg] \qquad (22)$$

6 Forces and moments on the elliptical cylinder

We shall now derive the formulae for force F(F_x, F_y) and moment M(M_x, M_y and M_z). Here x, y and z represent the components along the three coordinate axes respectively. The pressure normal to the surface of the elliptic cylinder is given by the linearized Bernoulli equation. Thus if p denotes the pressure, ρ the fluid density and g the acceleration due to gravity, then

$$p=\rho\Phi_t-\rho gz \quad \text{on } \xi=\xi_0 \qquad (23)$$

Then F_x and F_y are given by the formula

$$F_x^{(1)}=\int_S p\,(\hat{n}\cdot\hat{x})\;dS \qquad F_y^{(1)}=\int_S p\,(\hat{n}\cdot\hat{y})\;dS \quad \text{on } \xi=\xi_0 \qquad (24)$$

where S denotes the wetted surface of the body, $\hat{n}$ is the unit normal in the outward direction to the surface of the cylinder and dS is the elementary area.

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The solution of the first order forces can be written after considerable mathematical manipulation,as follows using $F_x = \epsilon F_x^{(1)}$ and $h = \epsilon H$ is the first order wave height. Thus

$$F_x = -\frac{8\rho gh \tanh k_1 d \sinh \xi_0}{k_1^2} \sum_{n=0}^{\infty} \Big[ce_{2n+1}(\alpha, q) ce_{2n+1}(0, q) \\ ce'_{2n+1}\left(\frac{\pi}{2}, q\right) Ve_{2n+1}(\xi_0, q) \cos(\beta_{2n+1} - \sigma t) \Big] \tag{25}$$

$$F_y = \frac{8\rho gh \tanh k_1 d \cosh \xi_0}{k_1^2} \sum_{n=0}^{\infty} \Big[se_{2n+1}(\alpha, q) se'_{2n+1}(0, q) \\ se_{2n+1}\left(\frac{\pi}{2}, q\right) Ue_{2n+1}(\xi_0, q) \cos(\gamma_{2n+1} - \sigma t) \Big] \tag{26}$$

The moments M_x and M_y about the x and y axes respectively, taken about the sea-bed $z = -d$ are

$$M_x = \int_S (z + d)\, p\, (\hat{n} \cdot \hat{y})\, dS$$

$$M_y = \int_S (z + d)\, p\, (\hat{n} \cdot \hat{x})\, dS \quad \text{on } \xi = \xi_0 \tag{27}$$

The twisting moment M_z about z-axis is

$$M_z = \int_S p\, (x\, \hat{n} \cdot \hat{y} - y\, \hat{n} \cdot \hat{x})\, dS \quad \text{on } \xi = \xi_0 \tag{28}$$

Following the same procedure as we did in obtaining F_x, we can write

$$M_x = \frac{8\rho gh(k_1 d \tanh k_1 d + \operatorname{sech} k_1 d - 1)}{k_1^3} \cosh \xi_0 \\ \sum_{n=0}^{\infty} \Big[se_{2n+1}(\alpha, q) se'_{2n+1}(0, q) se_{2n+1}\left(\frac{\pi}{2}, q\right) Ue_{2n+1}(\xi_0, q) \cos(\gamma_{2n+1} - \sigma t) \Big] \tag{29}$$

$$M_y = \frac{-8\rho gh(k_1 d \tanh k_1 d + \operatorname{sech} k_1 d - 1)}{k_1^3} \sinh \xi_0 \\ \sum_{n=0}^{\infty} \Big[ce_{2n+1}(\alpha, q) ce_{2n+1}(0, q) ce'_{2n+1}\left(\frac{\pi}{2}, q\right) Ve_{2n+1}(\xi_0, q) \cos(\beta_{2n+1} - \sigma t) \Big] \tag{30}$$

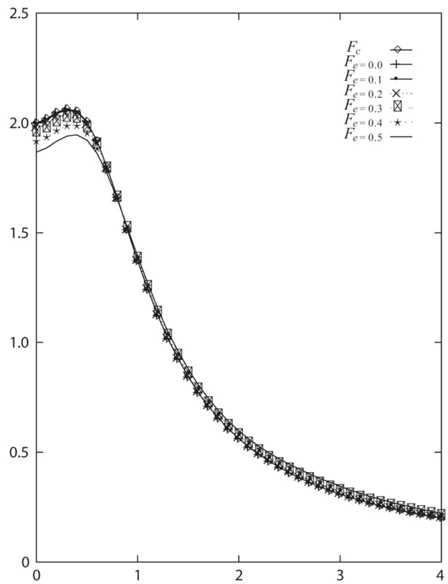

Figure 2: Comparison of the maximum non-dimensional horizontal forces for the elliptical and circular cylinder as a function of $k_1 a$ for $\alpha = 30^0$.

$$M_z = \frac{8\rho g h \tanh k_1 d}{k_1^3}$$

$$\sum_{n=0}^{\infty} \left[se_{2n+2}(\alpha, q) se'_{2n+2}(0, q) se'_{2n+2}\left(\frac{\pi}{2}, q\right) Ue_{2n+2}(\xi_0, q) \sin(\sigma t - \gamma_{2n+2}) \right] \tag{31}$$

Extensive numerical results for the forces and moments on an elliptical cylinder have been presented in graphical form for a wide range of relevant parameters by Chen and Mei [1]. As said earlier, calculation of the Mathieu functions and associated coefficients requires considerable computational effort, and, also, the convergence of the series expressions for the force and moment components is slow for large values of q.

7 Results and conclusions

The solution of Mathieu's equation led to a closed form expression for velocity potential on the surface of the elliptic cylinder in water of finite depth ; it is this velocity potential which is used to extract the formulae for the non-dimensional forces and overturning moments exerted by a linear progressive wave incident on the surface of the cylinder. Therefore, the values of maximum non-dimensional forces F(F_x, F_y) and overturning moment M_z will be used as the

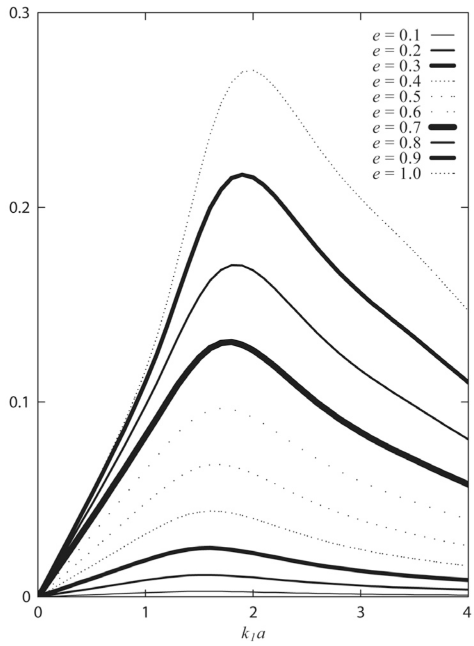

Figure 3: Maximum non-dimensional overturning moment as a function of $k_1 a$ M_z for $\alpha = 60^0$.

basis of discussion and comparison with the previous study of Williams [5]. The calculations of the aforementioned forces and moments require the evaluation of the special functions such as Mathieu functions which eventually necessitate the computations of the Bessel functions of integer order and positive real arguments. For simplicity these functions were computed by using the commercially available IMSL subroutine. A simple computer code is then developed to obtain the forces and overturning moments for the previous and present investigation.

Figure 2 shows the comparison of the maximum non-dimensional horizontal forces plotted against the non-dimensional wave number $k_1 a$ for the angles $\alpha = 30°$ and for eccentricities ranging from 0.0 to 0.5. The maximum of these forces occurs at zero wave number and decreases with increasing wave number $k_1 a$. In general, the forces appear to decline nearly exponentially with increasing wave numbers greater than 1.0. The effect of the angle of inclination α with respect to the incident wave seems to be more dramatic and clearly visible on the two components of the forces. It is found (not shown here) at the $\alpha = 30°$, the ratio of $\max|F_x|$ and $\max|F_y|$ is approximately 2 at $k_1 a = 0$ while it becomes 0.8 for $\alpha = 45°$ for $e = .1, .2, .3$. It is also found that the ratio of the $\max|F_x|$ to the $\max|F_y|$ decreases with increasing eccentricity. For example, at $k_1 a = 0$ and $\alpha = 30°$, the ratio of $\max|F_x|$ to $\max|F_y|$ is approximately 2.0 for $e = 0.1$ which becomes 0.5 for $e = 1.0$. For $\alpha = 45°$, max$|F_x|$ and max$|F_y|$ coincide for k_1a exceeding 2.0.

A sample curve depicted in Figure 3 shows the overturning moment max$|M_z|$, plotted against the non-dimensional wave number k_1a for $\alpha = 30°$. The figure contains max$|M_z|$ curves for different eccentricities ranging from $e = 0.1$ to $e = 1.0$. It is found that the values of max$|M_z|$ for the previous and present study are identical. The turning moment M_z is zero for zero wave number. Further it increases monotonically to attain a peak value at $k_1a \approx 1.9$ for most eccentricities and then starts declining. For $\alpha = 30°$, the peak value of max$|M_z|$ at $e = 1.0$ is observed to be an order of magnitude higher than its value at $e = 0.1$. This feature is loosely preserved for other angles of inclination as well. Similar trends are also observed for the peak values of max$|M_z|$ for other angles of incidence.

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