Analysis of tunnel compression wave generation and distortion by the lattice Boltzmann method

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Abstract

The finite difference lattice Boltzmann method was applied to investigate the generation of the compression wave produced when a high-speed train enters a tunnel and the distortion of the wave front as it travels in the tunnel. The discrete Boltzmann equation for the 3D39Q thermal BGK model is solved in three-dimensional space using a second-order Runge-Kutta scheme in time and a third order upwind finite difference scheme in space. The arbitrary Lagrangian-Eulerian formulation is applied to model the interaction of the moving train nose and the tunnel portal. Numerical calculations are carried out for axisymmetric trains with various nose profiles entering a long circular cylindrical tunnel with straight and stepwise flared portals. The generation of the compression wave can be described in terms of flow parameters along the train nose and the interaction with the tunnel portal. The train speed and the train to tunnel area dependence of the predicted wave strength are found to be in good agreement with an analytic linear prediction. The distortion of the compression wave front that travels within an acoustically smooth tunnel is consistent with the time-domain computation of the one-dimensional Burgers equation. The non-linear steepening is confirmed to be dependent on the initial steepness of the wave front, which is determined by the interaction of the train nose and the tunnel portal. The tunnel entrance with flared portals is shown not only to decrease the initial steepness of the compression wave front but also to counteract the effect of non-linear steepening.

Keywords: lattice Boltzmann method, high speed train, compression wave, micro-pressure wave, nonlinear steepening, aerodynamic noise.

1 Introduction

A high-speed train entering a tunnel compresses the air in front of it and generates the compression wave, which travels within the tunnel at the speed of sound toward the exit of the tunnel. The compression wave is reflected as an expansion wave at the exit of the tunnel being accompanied by the emission of an impulsive sound wave so-called 'micro-pressure wave' or 'tunnel bang' which causes the environmental annoyance in the nearby area (Ozawa et al [1]).

The strength of the micro-pressure wave is proportional to the steepness of the arriving compression wave, i.e. the gradient of the pressure across the wave front at the exit. The initial steepness of the wave front is determined primarily by the train speed and the geometries of train nose and tunnel portal. For a tunnel with acoustically smooth concrete slab tracks, and with the length comparable to the shock formation distance, which depends on the initial steepness of the compression wave, the wave front forms a shock wave due to the 'non-linear steepening'. This non-linear effect of acoustic propagation causes the strength of the micro-pressure wave to become comparable to the sonic bang from a supersonic aircraft. Fukuda et al [2] found that the compression wave in a tunnel longer than the shock formation distance once develops a shock profile, and then reverts by friction to a waveform with reduced steepness causing less subjective effects on the environment. Integrated numerical simulations will be necessary to investigate the propagation of compression wave and the radiation of micro-pressure wave such that the progressive characteristics of compression wave are interconnected to the original waveform.

A number of investigations on the generation and propagation of the compression wave have been carried out including scale model experiments by Aoki et al [3], theoretical analysis by Sugimoto and Ogawa [4] and by Howe [5], and numerical simulations by Ogawa and Fujii [6] and by Mashimu et al [7]. Howe has developed a linear theory of the compression wave. The head wave is expressed as a convolution product of the sources dependent on the nose shape and speed of the train, and an acoustic Green's function whose form depends on the geometry of the tunnel entrance. In the numerical calculations of Mashita et al, one-dimensional Euler's equation and the equation of state are solved to simulate the distortion of compressive wave front in the tunnel giving measured waveform as initial conditions.

The lattice Boltzmann method is now a very powerful tool of computational fluid dynamics. Tsutahara et al [8] have reported successful applications of the finite difference lattice Boltzmann method (FDLBM) to the direct numerical simulation of the sound generated a body immersed in fluids. Akamatsu et al [9] has demonstrated that the FDLBM is also a useful approach for studying problems in the non-linear acoustics. In this paper, we present direct numerical simulations of the generation of compression wave using the FDLBM. The arbitrary Lagrangian-Eulerian (ALE) formulation developed by Hirt et al [10] is applied to model the interaction of the moving train nose and the tunnel portal giving the local equilibrium distribution function in Eulerian form and moving the train and grids in Lagrangian manner.

2 Numerical method

2.1 Discrete BGK equation

The lattice Boltzmann model we use is a single relaxation time discrete BGK scheme on a three dimensional cubic lattice called D3Q39 model. The basic equation is written considering the arbitrary Lagrangian-Eulerian moving frame as:

$$\frac{\partial f_i}{\partial t} + (c_{i\alpha} - V_\alpha) \frac{\partial f_i}{\partial x_\alpha} - \frac{A}{\phi} c_{i\alpha} \frac{\partial (f_i - f_i^{\text{eq}})}{\partial x_\alpha} = -\frac{1}{\phi} (f_i - f_i^{\text{eq}}), \quad (1)$$

where f_i and f_i^{eq} are the velocity distribution function and the local equilibrium distribution function respectively, which represent the number density of particles having velocity $c_{i\alpha}$ at each lattice node. Subscript i represents the direction of particle translation and α indicates the Cartesian coordinates. V_α is the velocity of moving grid in ALE formulation, which is same as the speed of train translation. ϕ is the relaxation time and A is a constant of an additional term introduced to make the calculation of high Reynolds number flows fast and stable. These two parameters ϕ and A determine the viscosity of fluid.

The macroscopic variables of flow: density, velocity and internal energy are defined by the moment of velocity distribution function as follows:

$$\rho = \sum_i f_i, \quad (2)$$

$$\rho u_\alpha = \sum_i f_i c_{i\alpha}, \quad (3)$$

$$\frac{1}{2} \rho u^2 + \rho e = \sum_i \frac{1}{2} f_i c_{i\alpha}^2. \quad (4)$$

The local equilibrium distribution function f_i^{eq} is dependent on the macroscopic variables and is given by a polynomial of the flow velocity up to the third order. The pressure p and the sound velocity c_s are given by:

$$p = \frac{2}{3} \rho e, \quad (5)$$

$$c_s = \sqrt{\frac{10}{9}} e. \quad (6)$$

Eqn (1) is discretized in a finite difference scheme and the time integration is evaluated by the second-order Runge-Kutta scheme. The second and third terms on the left-hand side of eqn (1) are estimated using a forth-order central difference scheme added by a numerical viscosity term. The described LBM approach recovers the compressible Navier-Stokes equation. More details of the discrete BGK formulation and numerical procedure are referred to [8].

2.2 Numerical models

A cross section of the axisymmetric computational domain is shown in fig. 1, where the radius of tunnel normalizes the dimensions. The train is formed by a part of the inner boundary where a body fitted grid is employed. The no-slip boundary condition is applied to the train surface and slip boundary condition to the tunnel wall. The train travels in the negative x direction where the origin is at the centre of the tunnel entrance plane. The thick dotted line indicates the boundary of the moving frame being the grid nodes below the line to move with the same speed as the train.

The computations were made for three different train nose profiles: cone, paraboloid, ellipsoid and three different tunnel portals: unflared and two flared. The configurations of train nose profiles and tunnel portals are shown in fig. 2. The cross section of the train becomes uniform with constant area $A_0 = \pi h^2$ at a distance L from the nose of the train. The aspect ratio h/L of the nose is 0.2 for all profiles. The blockage A_0/A is 0.2 for the paraboloidal and ellipsoidal train, and is set to 0.12, 0.16, 0.2 and 0.25 for the conical train, where A is the cross-sectional area of the tunnel. The train Mach number is fixed to 0.25 for the paraboloidal and ellipsoidal train, but is changed to 0.15, 0.2, 0.25, and 0.3 for the conical train. The acoustic pressure was calculated at six microphone positions located on the tunnel wall (5) and in the outlet space (1). The microphone positions on the tunnel wall are $x/R = -10, -30, -50, -70$ and -90 where R is the radius of tunnel. The microphone position in the outlet space is at a distance $r/R = 10$ in the direction of 45 degree from the centre of the tunnel exit.

Figure 1: Cross section of axisymmetric computational domain.

Figure 2: Configuration of train nose profiles and tunnel portals.

3 Results and discussion

3.1 Flow field along the train nose

A train travelling at constant speed pushes aside the stationary air ahead and builds up a frozen flow field along the train. No sound wave is produced by the steady straight translation of train through a homogeneous environment. But the flow pattern changes and pressure waves are generated when the train nose sweeps past the variable geometry of the tunnel portal. Fig. 3 shows a variation of the flow field along a conical train nose at three relative positions to the tunnel portal. The pressure rise contour lines are shown in the left-hand side and the velocity vector plots in the right-hand side. Just before the train approaches the tunnel portal the air in tunnel starts to flow in the direction of train progression and a compressive head wave is observed to propagate as a plane wave with an initially small pressure rise fig. 3(a), when the train nose enters halfway into the tunnel air starts to flow out of portal fig. 3(b), and after the nose has passed into the tunnel and the uniform section of train passes the tunnel entrance plane, most of air flows out of the tunnel fig. 3(c). The hydrodynamic pressure is always the highest at the stagnation point of nose throughout the train progression.

Figure 3: Flow field along a conical train nose.

Spatial distributions of the pressure rise along the tunnel are plotted on a frame moving with a speed of sound in fig. 4. The parameters x_0 are the x-coordinates of train nose at each time step. The pressure ahead of the train nose gradually increases until the nose has passed into the tunnel. The pressure behind the nose section rapidly decreases to the ambient pressure at the tunnel entrance. Variations of axial flow velocity u in the axial and radial directions are shown in figs. 5 and 6. The air ahead of the nose flows forward to the train, but the air behind the nose flows backward toward the tunnel entrance except in the boundary layer enveloping the train. The branch is in the middle of nose.

Figure 4: Variation of pressure rise along the tunnel.

Figure 5: Variation of axial flow velocity along the tunnel.

Figure 6: Variation of axial flow velocity across the tunnel.

3.2 Initial wave profile at tunnel entrance

Fig. 7 shows a typical pressure rise waveform observed at a fixed position in tunnel ($x = -10$) and a time-distance diagram for the compression wave propagation and the train translation. The pressure at point “a” is same as the pressure at $x = 0$ when the nose passes the tunnel entrance plane. Air in the tunnel has been compressed before the train arrives at the entrance. The pressure at point “b” is that when the end of nose passes the entrance plane. The pressure increases rapidly between “a” and “b” due to the change of the cross-sectional area of the train. The head of train passes the fixed point at the time of “c”. The gradual increase of pressure between “b” and “c” is attributed to the growth of boundary layer on the train surface. The end of nose passes the point at the time of “d” and the pressure dives when the middle of nose has passed the point.

Howe has developed a formula for the pressure rise across the compression wave front based on a linear theory. The net pressure rise Δp across the wave front and the maximum pressure gradient are given by:

$$\Delta p = \frac{\rho_0 U^2}{1 - M^2} \frac{A_0}{A}, \quad (7)$$

$$\left(\frac{\partial p}{\partial t} \right)_{\max} \propto \frac{\rho_0 U^3 A_0}{R A}. \quad (8)$$

The formulas are valid for any nose profiles. The pressure rise $p/(\rho_0 U^2 A_0/A)$ and the pressure gradient $dp/dt/(\rho_0 U^3 A_0/RA)$ are plotted in fig. 8 against the non-dimensional retarded position of the train $U[t]/R$, where $[t]$ is the retarded time $t - x/c_0$, for train Mach numbers M between 0.15 and 0.3. The pressure rises in the

Figure 7: Wave profile observed at a fixed position in the tunnel and diagram for compression wave propagation and train translation.

Figure 8: Comparison of the pressure rise $p/(\rho_0 U^2 A_0/A)$ and the pressure gradient $dp/dt/(\rho_0 U^3 A_0/RA)$ for different Mach numbers $M = U/c_0$.

Figure 9: Comparison of the pressure rise $p/(\rho_0 U^2 A_0/A)$ and the pressure gradient $dp/dt/(\rho_0 U^3 A_0/RA)$ for different blockages A_0/A .

circles are referable to the pressures at “a” and “b” in fig. 7. It is found that the compression wave rise time is independent of Mach number, being equal to the effective transit time $\square 2R/U$ of the nose across the entrance plane of the tunnel. The pressure rise and the pressure gradient are plotted against $U[t]/R$ for the blockages A_0/A between 0.1225 and 0.25 in fig. 9. The pressure rise and the pressure gradient are essentially independent of blockage except the peak value of the pressure gradient. As a whole, these results confirm that the train speed and area ratio dependence on the pressure rise and the pressure gradient is consistent with the linear theory prediction for the wave initially produced at the tunnel entrance.

3.3 Comparison of nose profiles and tunnel portals

According to the linear theory [5], the pressure rise is expressed as a convolution product of the source term proportional to $\partial A_T(s)/\partial s$ and an acoustic Green's function, where $A_T(s)$ is the cross-sectional area of the train at distance s from the nose. The acoustic Green's function for the tunnel portal with an infinite flange is approximately given by a unit step function so that the initial waveform is primarily determined by $\partial A_T(s)/\partial s$. The area ratio A_T/A_0 and the rate of area change $\partial A_T/\partial s L/A_0$ are given for cone, ellipsoid and paraboloid respectively as:

$$\frac{A_T(s)}{A_0} = \frac{s^2}{L^2}, \frac{s}{L} \left(2 - \frac{s}{L} \right), \frac{s}{L}, \quad (9)$$

$$\frac{\partial A_T(s)}{\partial s} \frac{L}{A_0} = \frac{2s}{L}, 2 \left(1 - \frac{s}{L} \right), 1 \text{ for } 0 < s < L. \quad (10)$$

A comparison of the pressure rise and the pressure gradients is made for a conical, an ellipsoidal and a paraboloidal train in fig. 10. The peak values of the pressure gradients for the conical and ellipsoidal trains are almost the same, but the time of peak for the conical train is later by approximately 8 in non-dimensional time than the peak of ellipsoidal train. This time lag corresponds to the ratio L/U , where L is the length of nose and U is the speed of train. The pressure gradient of the paraboloidal train is flat and the value is smaller by approximately 30 % than other two trains.

Fig. 11 presents the pressure rise and pressure gradients predicted at $x/R = -20$ for the ellipsoidal train entering two and four step flared portals, comparing with the predictions for the unflared portal. The pressure rises for the flared portals are essentially linear with superposed ripples. It can be seen that the flared portal prevents the non-linear steepening of compression wave front.

Figure 10: Comparison of over-pressure and pressure gradient.

Figure 11: Pressure gradients for flared and unflared portals.

Figure 12: Variation of pressure gradients along the tunnel (cone).

3.4 Non-linear steepening in tunnel

Fig. 12 illustrates variations of the pressure rise and pressure gradients along the tunnel for the conical train. The waveform at $x/R = -90$ is deformed by the expansion wave reflected at the open end of tunnel. The peak values increase continuously as the distance from the tunnel entrance increases, demonstrating the deformation and steepening of the waveform in the tunnel. The non-linear deformation of waveform is verified by evaluating the following dimensionless form of Burgers equation (Cleveland et al [11]):

$$\frac{\partial P}{\partial \sigma} = AP \frac{\partial P}{\partial \theta} + \frac{1}{\Gamma} \frac{\partial^2 P}{\partial \theta^2}, \quad (11)$$

where $P(\sigma, \theta) = p/p_0$, p_0 is a reference pressure, $\sigma = x/\bar{x}$, $\bar{x}$ is the shock formation distance, $\theta = \omega\tau$, τ is the retarded time, Γ is the Gol'dberg number, and A is a coefficient. The LBM waveform at $x = -10$ is sampled as an initial waveform and the waveform evaluated by eqn (11) is compared with the LBM waveform at $x = -90$ in fig. 13. The agreement of both waveforms is excellent.

Figure 13: Comparison of LBM and Burgers prediction (cone).

Figure 14: Variation of pressure gradient as a function of distance from tunnel entrance for train Mach numbers M between 0.15 and 0.3 (cone).

The maximum pressure gradient $(\partial p / \partial t)_{\max}$ is plotted in fig. 14 against the distance from the tunnel entrance plane for the train Mach numbers between 0.15 and 0.3. The result confirms that the rate of non-linear steepening depends on the initial value of the maximum pressure gradient.

4 Conclusions

The FDLBM was applied to the direct numerical simulations of the compression wave generated by a high-speed train entering a tunnel and the distortion of wave front in the tunnel. Numerical calculations were carried out for axisymmetric trains with various nose profiles entering a circular cylindrical tunnel. The predicted pressure rise across the compression wave front and the maximum pressure gradient near the tunnel entrance are in good agreement with the linear theory. The distortion of the compression wave front travelling within an

acoustically smooth tunnel is consistent with the time-domain computation of one-dimensional Burgers equation. The non-linear steepening was confirmed to be dependent on the initial steepness of the wave front, and the tunnel entrance with flared portals was shown not only to decrease the initial steepness of the compression wave front but also to counteract the effect of non-linear steepening.

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