

# Numerical and experimental results for the warming of cylindrical timber segments

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## Abstract

As part of an industrial process in the forestry industry, pre-cut segments of debarked logs are *conditioned* by placing the wood segments in a warm water shower. For wood which is initially cold (or sometimes frozen) and has a relatively high moisture content, the conditioning process results in a net increase in temperature and change in moisture content. As a porous media, wood has a fairly complex structure due to the orientation and sizes of the cells which make up the fibers. In addition, the moisture content of the recently debarked log segments is relatively high, the moisture being almost completely composed of liquid water trapped by capillary forces. Simplistic models which assume a constant moisture content have previously been used to predict conditioning time. However, as is often the case in reality, the complexity of the problem is such that more accurate modelling is desired.

To learn more about the conditioning process, preliminary experiments were conducted using a data logger and an array of fifteen thermocouples placed radially at three points on a log segment which was then put through a full conditioning cycle. Theoretical model equations are presented, which take into account both the liquid and vapour components of the water, and spatial averaging is used to create a system of nonlinear partial differential equations. The problem is then simplified through some basic assumptions to pose a set of coupled differential equations for energy and moisture transport in one spatial dimension. A finite difference numerical method is used to solve the initial boundary value problem for the coupled system of nonlinear partial differential equations, and some numerical results are compared to experimental data for the one dimensional problem. Limitations of the model are identified and discussed.

*Keywords: multiphase flow, porous media, water transport, finite differences.*



## 1 Introduction

The problem studied herein arises from an industrial application in the manufacturing of plywood. Of direct concern to industry is finding a quick and simple method to predict the amount of time required to warm pre-cut and debarked cylindrical wooden log segments from their ambient temperature outside to a desired production temperature. This process is known as *conditioning* and can be achieved in many ways, for example, submersing the segments in a hot water bath or exposure to hot steam. In this study, conditioning is achieved by placing the wood in a continuous shower of warm water, and waiting for a pre-determined time to move the wood onto the next stage in the production process. As a porous media, fluid within the cell structure can move from cell to cell under the influence of temperature and concentration gradients, and therefore the transport of water must be considered as an integral part of the conditioning process.

The specific problem considered is therefore to create a model for the reheating of a cylinder of wood through partial contact of its outer boundary with very humid air. In the next section, a set of governing equations are posed which describe the reheating of wood in this particular circumstance. The model takes into account the porous nature of wood, and the multiphase (liquid and gas) flow of water within the porous media in order to obtain a set of partial differential equations for the volume averaged temperature and moisture content. In section 3, the equations numerical solutions of the equations are compared to experimental data gathered on site. Some concluding remarks follow, and limitations of the model are discussed.

## 2 Model equations

A first attempt at modelling the heating of a homogeneous cylinder is typically achieved through the application of a linear form of the heat equation,

$$\frac{\partial T}{\partial t} = \nabla \cdot k_q \nabla T \quad (1)$$

where  $T$  is the temperature,  $t$  time, and  $k_q$  a heat conduction coefficient. Equation (1) has been well-studied [1], although there are several factors which are important when studying the warming of wood which require a more delicate and thoughtful approach. For example, the coefficient  $k_q$  can depend strongly on moisture content, and to a lesser extent, temperature, tree species, and spatial position within the wood [2]. Equation (1), coupled with an analogous moisture diffusion equation, has been used to model wood heating [3] and the non-isothermal drying of dimensional lumber [4]. Other limitations are imposed by assuming equation (1), particularly regarding moisture content. Heating and surface evaporation can result in the redistribution of fluid within the wood. Indeed, starting from the time of harvest, wood immediately begins to lose moisture, a process which is accelerated with de-barking. The humid environment employed during conditioning acts to reduce surface evaporation but still allows for the formation of pressure gradients which result in a nonhomogeneous moisture distribution.



Whitaker [5] completed a thorough characterization of the transport of energy and mass in general porous media, which was later applied to dimensional lumber drying by Spolek [4]. Whitaker's approach [5] considered the phases of water and the solid wood separately. The temperature  $T$  of the wood is modelled with a linear enthalpy approximation and Fourier's law of heat conduction as a generalization of equation (1).

$$\rho_{\sigma}(c_p)_{\sigma} \left( \frac{\partial T_{\sigma}}{\partial t} \right) = k_{\sigma} \nabla^2 T_{\sigma} + \Phi_{\sigma}. \quad (2)$$

In equation (2), and in later equations, the standard symbols are used:  $\rho$  for density,  $c_p$  heat capacity,  $k$  thermal conductivity coefficient, and  $\Phi$  for heat sources. The subscript  $\sigma$  is used for all of these variables to denote the properties within the solid wood.

The liquid phase obeys a similar continuity equation, with the added complication of transport, and can be stated (with subscripts  $\beta$  denoting liquid phase) as

$$\rho_{\beta}(c_p)_{\beta} \left( \frac{\partial T_{\beta}}{\partial t} + \mathbf{v}_{\beta} \cdot \nabla T_{\beta} \right) = k_{\beta} \nabla^2 T_{\beta} + \Phi_{\beta}. \quad (3)$$

In equation (3) a linear relationship between enthalpy and temperature is again used, as is Fourier's law of conduction within the liquid.

For the gas phase, the temperature of the water vapour and the existing inert gases (such as air) are taken into account to give a generalization for the gas phase, written with a subscript  $\gamma$  as

$$\rho_{\gamma}(c_p)_{\gamma} \left( \frac{\partial T_{\gamma}}{\partial t} + \mathbf{v}_{\gamma} \cdot \nabla T_{\gamma} \right) = k_{\gamma} \nabla^2 T_{\gamma} + \Phi_{\gamma} - \nabla \cdot \left( \sum_{i=1}^{i=N} \rho_i \mathbf{u}_i (c_p)_i T_{\gamma} \right), \quad (4)$$

where the last term represents the contribution due to the existence of  $N$  inert gases with diffusion velocity  $\mathbf{u}_i$  and heat capacity  $(c_p)_i$ .

Equations (2), (3), and (4) are local in nature, and are standard expressions in the transport of fluids with the inclusion of diffusion and a source/sink term ( $\Phi$ ). To obtain relationships between the bulk properties of the wood, the equations must be averaged over a given volume. For example, for a volume element  $V$ , the average temperature of the solid phase is given by

$$\langle T_{\sigma} \rangle = \frac{1}{V} \iiint_V T_{\sigma} dV. \quad (5)$$

Each of equations (2), (3), and (4) are integrated to create volume averaged equations, which are subsequently manipulated using integration by parts theorems to bring in various phase boundary conditions into the problem. The individual equations are then added together to create a total thermal energy equation, for the averaged temperature  $\langle T \rangle$  [5]

$$\begin{aligned} & \langle \rho \rangle c_p \frac{\partial \langle T \rangle}{\partial t} + [\rho_{\beta}(c_p)_{\beta} \langle \mathbf{v}_{\beta} \rangle + \langle \rho_{\gamma} \rangle^{\gamma} \langle (c_p)_{\gamma} \rangle^{\gamma} \langle \mathbf{v}_{\gamma} \rangle] \cdot \nabla \langle T \rangle + \Delta h_{vap} \langle \dot{m} \rangle \\ & = \nabla \cdot \mathbf{K}_{eff}^T \nabla \langle T \rangle + \langle \Phi \rangle. \end{aligned} \quad (6)$$



The process leading to the derivation of equation (6) is quite detailed due to the consideration of all of the phases, and the details may be found elsewhere [5]. In equation (6) a few shorthand terms have been introduced: the superscript  $\gamma$  denotes an average over the portion of the volume element in relation to that phase,  $\langle \Phi \rangle$  denotes a combined source term, and  $\langle \dot{m} \rangle$  denotes the mass rate of vaporization per unit volume. In addition, the enthalpy of vaporization per unit mass at temperature  $\langle T \rangle$  is given as  $\Delta h_{vap}$ , and being a non-homogeneous material,  $K_{eff}^T$  is a second order tensor which encompasses the thermal diffusivity in each of the three spatial dimensions.

A second equation can be created for the moisture content by applying an approach analogous to the derivation of equation (6). Again, the details are left to Whitaker [5], and the result is stated for the saturation,  $S$ , a quantity which represents the mass fraction of moisture in the form of liquid and vapour to the fully saturated mass.

$$\phi \frac{\partial \langle S \rangle}{\partial t} + \nabla \cdot \left( \langle \mathbf{v}_\beta \rangle + \frac{\langle \rho_1 \rangle^\gamma \langle \mathbf{v}_\gamma \rangle}{\rho_\beta} \right) = \nabla \cdot \left[ \frac{\langle \rho_\gamma \rangle^\gamma}{\rho_\beta} D_{eff}^{(1)} \nabla \left( \frac{\langle \rho_1 \rangle^\gamma}{\langle \rho_\gamma \rangle^\gamma} \right) \right]. \quad (7)$$

In equation (7), the effective diffusivity  $D_{eff}^{(1)}$ , is a second order tensor which encompasses diffusivity of the vapour in each of three spatial dimensions,  $\rho_1$  is the vapour density, and  $\phi$  is the porosity or volume fraction of the void space in the wood.

To close the system, the momentum equations for the liquid and gas phases are used together with Darcy's law to relate the flow rate to the pressure gradient. The pressure  $P$  is then introduced in place of velocity, with the introduction of permeability constants  $K$  (with appropriate subscripts) The resulting two equations, neglecting heat sources and sinks  $\langle \Phi \rangle$ , are stated as

$$\begin{aligned} \langle \rho \rangle c_p \frac{\partial \langle T \rangle}{\partial t} - \left[ \frac{K_\beta \rho_\beta (c_p)_\beta}{\mu_\beta} (\nabla \langle P_\beta \rangle^\beta - \rho_\beta \mathbf{g}) + \frac{K_\gamma \langle \rho_\gamma \rangle^\gamma \langle (c_p)_\gamma \rangle^\gamma}{\mu_\gamma} \right. \\ \left. \times (\nabla \langle P_\gamma \rangle^\gamma - \langle \rho_\gamma \rangle^\gamma \mathbf{g}) \right] \cdot \nabla \langle T \rangle + \Delta h_{vap} \langle \dot{m} \rangle = \nabla \cdot K_{eff}^T \nabla \langle T \rangle, \end{aligned} \quad (8)$$

and

$$\begin{aligned} \phi \frac{\partial \langle S \rangle}{\partial t} - \nabla \cdot \left[ \frac{K_\beta}{\mu_\beta} (\nabla \langle P_\beta \rangle^\beta - \rho_\beta \mathbf{g}) + \frac{\langle \rho_1 \rangle^\gamma K_\gamma}{\rho_\beta \mu_\gamma} (\nabla \langle P_\gamma \rangle^\gamma - \langle \rho_\gamma \rangle^\gamma \mathbf{g}) \right] \\ = \nabla \cdot \left[ \frac{\langle \rho_\gamma \rangle^\gamma}{\rho_\beta} D_{eff}^{(1)} \nabla \left( \frac{\langle \rho_1 \rangle^\gamma}{\langle \rho_\gamma \rangle^\gamma} \right) \right]. \end{aligned} \quad (9)$$

Equations (8) and (9) contain several coefficients which must be determined prior to solving the equations. By applying a mechanistic model of wood structure, the properties of wood may be used to develop theoretical relationships and values for many of the parameters to decrease the dependence on experimental parameters [2, 4]. As in the development of the equations, the technical details call

for a more thorough examination of their derivation; however, for brevity, only the impact of the simplifications will be used herein with the details referred to where appropriate. For example, the volume fraction  $\phi$  can be determined by an analytic dependence on cell parameters [4].

The liquid and gas phase pressures remaining in equations (8) and (9) can be removed by assuming the gas phase acts as an ideal gas, and that the capillary pressure  $P_c$  is the difference between the gas phase pressure and liquid phase pressure, [5, 4]. The liquid phase pressure is therefore expressed as a function of the gas phase pressure and the capillary pressure,  $\langle P_\beta \rangle^\beta = \langle P_\gamma \rangle^\gamma - \langle P_c \rangle^c$ . The liquid pressure gradient can then be expressed by the following linear expansion,

$$\begin{aligned} \nabla \langle P_\beta \rangle^\beta = & \left( \frac{\partial \langle P_1 \rangle^\gamma}{\partial T} + \frac{\partial \langle P_2 \rangle^\gamma}{\partial T} - \frac{\partial \langle P_c \rangle^c}{\partial T} \right) \nabla \langle T \rangle \\ & + \left( \frac{\partial \langle P_1 \rangle^\gamma}{\partial S} + \frac{\partial \langle P_2 \rangle^\gamma}{\partial S} - \frac{\partial \langle P_c \rangle^c}{\partial S} \right) \nabla S. \end{aligned} \quad (10)$$

Analytic expressions for the partial pressures and capillary pressures in equation (10) are given by Spolek [4] as functions of temperature, saturation, and number of constants. In the interest of writing equation (10) in a more compact form, the following substitutions are made,

$$\nabla \langle P_\beta \rangle^\beta = -(C_{T,c} - C_{T,\gamma}) \nabla \langle T \rangle - (C_{S,c} - C_{S,\gamma}) \nabla \langle S \rangle. \quad (11)$$

In addition, a dimensionless temperature  $Q$  is used ( $M$  is already dimensionless definition) such that  $Q = (T - T_0)/(T_\infty - T_0)$ , where  $T_0$  is the average initial internal temperature of the log, and  $T_\infty$  is the temperature of the bathing fluid. Since the saturation  $S$  does not include the hygroscopically bound water, it will be substituted for with the moisture content  $M$  through the relation  $S = (M - M_{fsp})/(M_{max} - M_{fsp})$ , where the subscript *fsp* denotes the fiber saturation point, and the subscript *max* refers to the maximum possible moisture content the wood can contain. The fiber saturation point is a species dependent parameter which is a measure of the moisture content due to hygroscopically bound water only.

In addition, as wood dries, surface tension forces the small valve-like openings which connect neighbouring cells, known as pits, to be forced shut. A chemical bond between the valve and pit opening forms, effectively sealing off the pit opening to gas transport, causing the gas phase to become trapped and permitting the assumption of a gas phase permeability very close to zero, ( $K_\gamma = 0$ ) [2]. Although the bulk gas phase transport is neglected, vapour transport continues as a diffusive process which is captured in a diffusion coefficient denoted  $D_m$ , leading to the following simplification,

$$\langle \rho_\gamma \rangle^\gamma D_{eff}^{(1)} \nabla \left( \frac{\langle \rho_1 \rangle^\gamma}{\langle \rho_\gamma \rangle^\gamma} \right) = \rho_\beta D_m \nabla M, \quad (12)$$

where  $M$  is the moisture content, and  $D_m$  is a combined vapour and bound water moisture diffusion coefficient.



Employing the above simplifications, and neglecting transport due to gravity along with equations (12), and (10), permits equations (8) and (9) to be written as a more simplified set of transport equations,

$$\begin{aligned} \langle \rho \rangle c_p \frac{\partial \langle Q \rangle}{\partial t} + \left[ \frac{\rho_\beta (c_p)_\beta K_\beta}{\mu_\beta} ((C_{T,c} - C_{T,\gamma}) \Delta T \nabla \langle Q \rangle \right. \\ \left. + (C_{S,c} - C_{S,\gamma}) \frac{1}{\Delta M} \nabla \langle M \rangle) \right] \cdot \nabla \langle Q \rangle = \nabla \cdot K_q \nabla \langle Q \rangle, \end{aligned} \quad (13)$$

and

$$\begin{aligned} \frac{\phi}{\Delta M} \frac{\partial \langle M \rangle}{\partial t} + \nabla \cdot \left[ \left( \frac{K_\beta}{\mu_\beta} (C_{T,c} - C_{T,\gamma}) \right) \Delta T \nabla \langle Q \rangle \right. \\ \left. + \left( \frac{K_\beta}{\mu_\beta} (C_{S,c} - C_{S,\gamma}) \right) \frac{1}{\Delta M} \nabla \langle M \rangle \right] = \nabla \cdot D_m \nabla \langle M \rangle. \end{aligned} \quad (14)$$

### 3 Results

the thermal conductivity can be reduced to a scalar quantity  $k_q$ , and A moisture diffusion coefficient,  $D_m$ , is introduced in an analogous way to the thermal conductivity derivation [4].

This section considers the restriction of equations (13) and (14) to cylindrical polar coordinates, with only the radial motion considered, such that  $Q = Q(r, t)$  and  $M = M(r, t)$ . The equations are simplified further by suppressing the volume average bracket notation, and using the notation  $C^T = (C_{T,c} - C_{T,\gamma})$  and  $C^S = (C_{S,c} - C_{S,\gamma})$ . In addition, the second order tensors  $K_{eff}^T$  and  $D_m$  can be reduced to scalar quantities  $k_q$  and  $d_m$  respectively, when considering only the radial direction. The resulting equations are expressed as

$$\rho c_p \frac{\partial Q}{\partial t} + \left[ \frac{\rho_\beta (c_p)_\beta K_\beta}{\mu_\beta} (C^T \Delta T \frac{\partial Q}{\partial r} + C^S \frac{1}{\Delta M} \frac{\partial M}{\partial r}) \right] \frac{\partial Q}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} r k_q \frac{\partial Q}{\partial r}, \quad (15)$$

and

$$\frac{\phi}{\Delta M} \frac{\partial M}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} r \frac{K_\beta}{\mu_\beta} \left[ C^T \Delta T \nabla Q + C^S \frac{1}{\Delta M} \nabla M \right] = \frac{1}{r} \frac{\partial}{\partial r} r d_m \frac{\partial M}{\partial r}. \quad (16)$$

Equations (13) and (14) are applied to model a symmetric simplification of log conditioning, considering only time-dependent radial transport of heat and moisture. The initial boundary value problem is solved in the domain  $t > 0$  and  $0 < r < R$ . The boundary conditions at  $r = 0$  are the standard symmetry conditions of zero flux. The outer boundary conditions at  $r = R$  are somewhat more complex, as a balance of energy and mass must be maintained at this point causing the latent heat of evaporation to be included to derive a generalized Neumann boundary condition which couples moisture, temperature and the flux at the surface.



The thermal boundary can be approximated by the standard convective heat flux boundary condition,  $\dot{q} = h_q(T(R, t) - T_\infty)$  [6], where  $h_q$  is the surface thermal transfer coefficient, and  $T_\infty$  is the temperature of the bathing fluid. The horizontal stacking of the cylindrical log segments in the conditioning tunnel give rise to horizontal voids between neighboring log segments. By approximating the voids as cylindrical, and the flow of the humid air in the voids to be in the laminar range, the surface heat transfer coefficient  $h_q$  can be expressed as  $h_q = (k_\gamma/D)0.023Re_D^{0.80}Pr^{0.33}$ , where  $k_\gamma$  is the thermal diffusion coefficient of humid air,  $D$  is the diameter of the void,  $Re$  is the Reynolds number of the flow, and  $Pr$  is the Prandtl number of the humid air.

The boundary condition for the moisture content should ideally be analogous to the thermal flux condition. Unfortunately, a mass flux boundary which does not result in an ill-posed problem has not yet been found and will therefore be approximated with a Dirichlet condition,  $M(R, t) = M_0$ , where  $M_0$  is an appropriately chosen initial moisture content at the surface. Although a Dirichlet moisture boundary results in a somewhat unphysical moisture boundary condition, it will suffice as a first approximation until appropriate mass flux boundaries at the surface can be derived.

With the initial and boundary conditions specified, a finite difference numerical approach was taken to find numerical solutions. A forward in time central in space scheme was chosen, with some care employed to obtain second order spatial approximations for the Neumann boundary conditions. An exciting feature of this research program was the ability to conduct measurements of log conditioning, and data was collected for several experiments. Thermal sensors were placed in the log in a straight line along a ray outwards from the center with one at  $r = 0$  cm (core), one at  $r = R$  (surface), and three more positions between the core and the surface. The data from the temperature sensors was recorded over a period of several hours during conditioning. Moisture content was not measured during the experiment, however a rough estimate of initial and final moisture content was measured by removing samples for controlled measurement in the laboratory. The main sources of error were in the temperature measurement and placement of the thermocouples, and the lack of precision in moisture content measurements.

A selected result is displayed in Figure 1, which depicts temperature versus time giving a comparison of the model to experimental temperature data 30 cm from the end, and at both the core and 10 cm radially from the core. The thin dotted lines represent the placement error of the thermocouples.

## 4 Conclusion

The modelling of any industrial process can be plagued by oversimplification which causes the problem to be either trivial or too far removed from reality to be of much use. In this case, the modelling of log conditioning is a complicated process involving a consideration of the vapour and liquid phases of moisture moving within a the porous wooden cellular network. Through volume averaging, a system of differential equations, derived from a lumber drying model, were applied and



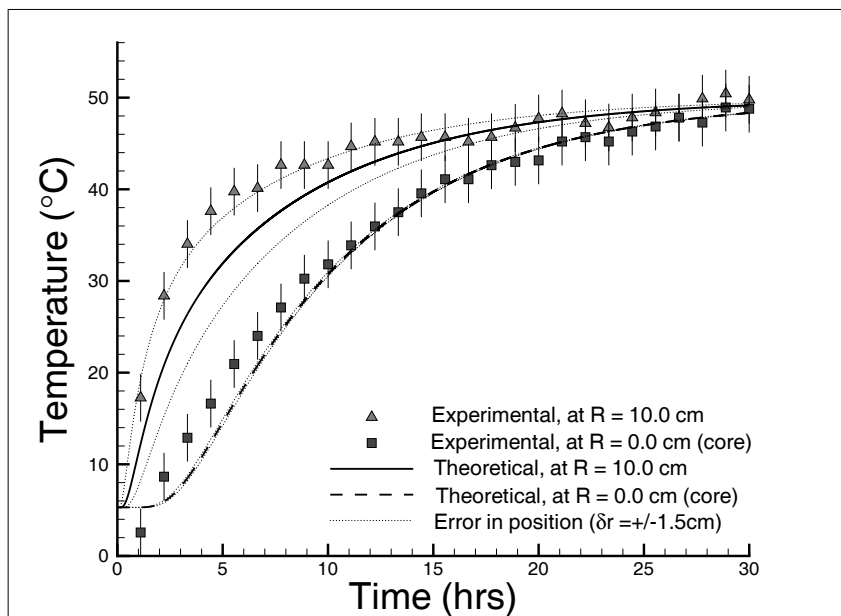


Figure 1: Comparison of theoretical model to experimental data for Douglas Fir, where  $T_0 = 5.3^\circ\text{C}$ ,  $M_0 = 0.80$ ,  $R = 15\text{ cm}$ ,  $Z = 30\text{ cm}$ .

found to model the experimental results relatively well within experimental error. Some underestimation of the temperature is seen which could be due to heating from the end of the log, which is neglected in the one dimensional problem. In addition, the Dirichlet boundary condition imposes a relatively unphysical moisture boundary which could cause the model to converge to an incorrect steady-state. Ongoing research will assist in probing the dependence of various types of boundary conditions, and generalizing the problem to more than one spatial dimension to account for these minor discrepancies.

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