

## INITIAL STUDIES OF THE DEFORMATIONS OF VAL D'ARAN CHURCHES

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### ABSTRACT

In the region of Val d'Aran, there is an ensemble of Romanesque churches with very particular formal and constructive characteristics. These buildings, most of which were built during the 12th century, have suffered various alterations throughout history. Their capacity to deform in many cases has pushed the balance of the masonry structure to the limit, forcing the introduction of stabilizing elements.

The aim of this investigation is to evaluate the construction and equilibrium conditions of these highly deformed shapes. This article introduces the context of the churches of Val d'Aran from a wide point of view and addresses the formal characteristics of some of the most relevant examples of basilica floor plan constructions.

An assessment of stability is performed in the church of Santa Eulària d'Unha, which has one of these basilica floor plans, with large deformation of the masonry structure. The methodology is based on graphic statics within the theoretical framework of limit analysis.

*Keywords: limit analysis, masonry, Unha, Val d'Aran.*

### 1 GEOGRAPHICAL, HISTORICAL AND CULTURAL FRAMEWORK

The Aran Valley, Valle de Arán in Spanish or Val d'Aran in Aranese, the native language, is located in the north of the Pyrenees (province of Lleida, Spain). It is bordered by France (Department of Haute-Garonne) to the north, the region of Ribagorza (province of Huesca) to the southwest and the Catalan regions of Alta Ribagorza and Pallars Sobirà (province of Lleida) to the south and east. The majority of the valley is located in the Atlantic basin. It has an extension of 634 km<sup>2</sup> and 30% of its surface is over 200 m in altitude.

Since ancient times, the human relations, both commercial and cultural, with the north following the natural course of the Garona River have been very intense, especially with the French regions of Comminges (Department of Haute-Garonne) and Couserans (Department of Ariège). Proof exists in the archaeological remains of the prehistoric and Roman times. It was not until the Middle Ages that political relations began with the states located on the southern gradient of the Pyrenees. Between the 11th and the 15th centuries, a large historical process occurred in which in a progressive way it became part of Aragon's kingdom, then part of the principality of Catalonia and, finally, after the vanishing of the Aragonese crown, part of Spain.

This process was opposed by the French monarchy, who on numerous occasions occupied the Aran Valley. The longest and most transcendent of these occupations occurred after the invasion of November 1283. The King of France claimed his rights over the territory, and the Aran Valley belonged to France for more than 15 years. For this reason, a long dispute occurred, and when it was not solved, Aran was left in the hands of the king of Majorca. The Poissy Treaty, on 26 April 1313, ended the conflict, and Philip IV of France restored Aran to the Crown of Aragon. Among these struggles, the Aranese knew how to negotiate and enforce their rights and privileges. Not long after their return to the Aragonese Crown, on 22 August

1313, King James II of Aragon established the privilege of 'Era Querimònia', which is a written compilation of their usages, rights and customs. However, the Church continued in the French bishopric of Comminges until its disappearance, and in 1804, it was annexed to the diocese of Urgell (Catalonia), to whom it now belongs.

In this context, rich and varied natural and cultural heritages have been preserved, extremely concentrated in the valley of no more than 40 km in length. Thirteen small villages are located in this natural environment, which has a high landscape value, arranged in nine municipalities. It has an interesting ethnological heritage; the traditional stock economy is concentrated in the household as the economic, social and political unit. There are hydraulic flour mills, washing places, and, based on the industrial heritage, lime ovens, wool factories, iron and zinc mines and, most recently, large hydroelectric plants. No less important is its immaterial heritage, which is highlighted by the Aranese language, a variant of the Occitan language, spoken in the Aran Valley, and numerous festivities and traditions.

## 2 ARCHITECTONIC AND ARTISTIC HERITAGE

The Aran Valley has retained a large and concentrated group of parochial churches and chapels of Romanesque and Gothic style. In addition, ruins of various castles, some of which emerged to protect the parochial churches, and watch and defensive towers can be found.

Unfortunately, there is a lack of historical documentation to date these buildings. Up to now, the chronologies taken as reference are based on the study of the materials, the construction techniques and the style characteristics, comparing them with other monuments. The oldest chronologies date from the beginning of the 20th century and are the result of two scientific expeditions. The first one was led by Lluís Domènech i Muntaner in the year 1904, from the architecture school of Barcelona, the materials of which were published by Granell and Ramon [1], and the second one, in 1907, was the 'Missió arqueològica' of Josep Puig i Cadafalch, Gudiol and company, the materials of which were published by Alcolea [2]. The result from this trip is the synthesis and interpretation of Puig i Cadafalch [3] on the characteristics of the Romanesque Aranese architecture. After, the Aran Valley fell into oblivion, unlike the Vall de Boí, which is easier to access. It was not until the works of Sarrate Forga [4–6] and the encyclopaedic publication *Catalunya Romànica* [7] that a more complete and comprehensive revision of contents was achieved.

Despite these shortcomings, it must be recognized that in the last two decades knowledge about the Aran Valley has significantly increased, with specific studies and the results of several interventions carried out with some movable and immovable goods, which have contributed to the generation of new information on many topics.

During the 11th and 13th centuries, an original and attractive Romanesque art flourished in the valley, far from the main creation centres. It conveys archaic elements with other mid- or even late-Romanesque characteristics. There are signs that relate the origin of this flourishing to the renewal of Comminges during the episcopate of Bertrand de l'Isle (1083–1123). He was canonized in 1308, with his name given to the episcopal Seo of Comminges.

Thirty-five churches whose constructions date from this period are preserved standing or with significant remains. The most antique group is inscribed in the first Romanesque style, although in many cases they are late constructions, fully built in the 12th century, as in the neighbouring Valley of Boí. They are characterized by the use of small ashlar roughed down by hammer; the total absence of sculpt decoration except for toothed frieze, blind arches and pilaster strips; cut tufa stone, which articulates and gives life to the exterior surfaces of the walls, especially in the apses; the absence in some of the construction of vaults; the use of

calcareous tufa in arches and vaults; and the simplicity and narrowness of the overtures, both doors and windows.

Santa Maria de Cap d'Aran is a first prototype church, and its construction should have started in the 11th century. After, all the great Romanesque churches in the valley during the 12th and 13th centuries adopted the basilica floor plan with three naves and four structural vanes. The church of Cap d'Aran is the only one with a small crypt. These basilica buildings were covered in the central nave with a barrel vault and in the lateral naves with a quarter of circle vaults. In the late building of St. Andrèu of Salardú, transitioning to Gothic, the coverage of the nave is made with cross vaults. In these buildings of the basilica floor plan, the pilasters that sustain the vaults and divide the naves are usually circular. This is a common characteristic when compared with the Romanesque architecture of the neighbouring Valley of Boí.

During the 12th century, even during the first third of the 13th century, Romanesque buildings were still being built but with better achieves on exterior surfaces, through greater and more regular rigs, set out in horizontal courses, cut regularly and with a smoother surface. Externally, a single-slate roof with double slope covers the three naves. The simultaneous presence of two types of bells is characteristic: a belfry wall in the eastern gable and a robust bell tower are usually attached. However, few cases exist where the tower is Romanesque. Many of the bell towers were completed at a later time, in Gothic style, or even later. The slender pyramidal slate spire that covers them gives considerable uniqueness to these towers.

They have one or two entrances that are usually located in the lateral façades. From the 12th century, these doors became generously sculpted decoration on the tympanums, archivolt and capitals, usually the work of some local artist. Despite the power of the Romanesque tradition in the end of the 13th century and even more in the 14th century, new ideas of Gothic artistic and architectural trends arrived to the valley.

The beginnings of the new style coincided with the French (1283–1295) and Majorca (1295–1313) occupations. The French style is obvious in the bell tower windows of Vilac or in the decoration of the exterior archivolt of the main entrance of the church of St. Miquèu de Vielha, as explained by Español [8]. In addition, in this church, a new type of monumental door is found, in which the longitudinal direction of the pointed arches is superimposed, which is a timid tendency to the disappearance of the tympanum and the use of not only the space of the tympanum but also of the archivolt as a support for an abundant sculpture. Subsequently, a simpler typology devoid of sculptural decoration was adopted. With architectonic solutions from the so-called meridional Gothic, some of the Romanesque buildings were finished or restored. In Salardú, the central nave of the church is covered with crossed vaults, and the western wall is finished with a magnificent ogival window. In many other Romanesque churches, the importance of light in this period forced the opening of wider windows, especially in the western facade. In other cases, larger restorations and extensions were taken on or new buildings were constructed. These new constructions acquired the formula of a unique nave, but maintaining a tripartite sanctuary, a consequence of the deep roots of the basilica floor plan in native builders.

The Aran churches, according to the intense conflicts of the period, became very fortified enclosures defended by a wall flanked with towers in this time. In addition, the church, this enclosure protected some annexed buildings and the cemetery. The bell towers were provided with loopholes and battlements. Between the 15th and early 16th centuries, this evolution of the Aranese bell towers towards a fully military appearance culminated in the Gothic towers of octagonal section of Salardú and Vielha.

### 3 THE BASILICA FLOOR PLAN CHURCHES OF ARAN VALLEY

The interest in these Romanesque structures took the Escola Tècnica Superior de Arquitectura of Rovira i Virgili University to study them in 2012–13 and 2013–14 courses. Three campaigns were conducted. The first occurred in winter (2012), and the church of Santa Maria de Arties was surveyed, where Bassegoda (1972) noticed the large deformations of the structure. In the second campaign in the following spring (2013), Santa Maria d'Arties was surveyed again in greater detail. A third campaign was carried out to survey the churches of Vilamos, Salardú, Bossost and Unha. As a result of these three campaigns, the floor plan and main cross sections of these churches were laid out. The general characteristics are as follows. The churches have a basilica floor plan with three naves, two laterals with similar height and one higher central nave. All the churches, except Vilamos, have a Romanesque apse, and all of them have a bell tower. Finally, the most important characteristic is that all the churches have a central barrel vault and half barrel vault at the lateral naves.

The church of Santa Maria d'Arties can be defined as a rectangular space of approximately  $19.60 \times 13.8$  m until the apsidioles (Fig. 1). This measure is only indicative because the great deformations make it impossible to accurately set the general building measures. The construction is raised with a typical east–west orientation, with a central nave and two collaterals, each with four bays and a span of approximately 5 m.

The central pillars support the former arches and are located under the gathering of the central barrel vault and the lateral half vaults. In the western side of the church, a wooden choir is found over the access bay. The heading is currently finished with a semi-circular apse, reconstructed during the last few years, and the two original apsidioles. All of them are covered with hemispherical domes. The naves present a typical deformation pattern according to the structural arrangement. Vertical supporting elements, especially the pillars, have got out of plumb because of the thrusts of vaults. The deformation has been measured, in degrees ( $\alpha$ ) at the wall or column base. Arties have  $0^\circ$  in the north wall, one and five in the inner columns, and  $4^\circ$  in the south wall. The horizontal displacement of the upper part of the northern wall measures 0.03 m, and the wall measures 0.98 m in width at the base. If the difference in height of the keystone of the vaults is related with the width of walls and their displacements, Arties is the most deformed church, with a difference of 1.91 m in height (measured at the keystone) between the lateral vault and the central vault. The span of the central vault is 0.21 m larger at the springing than the distance between the supports at the base. The height and width of the

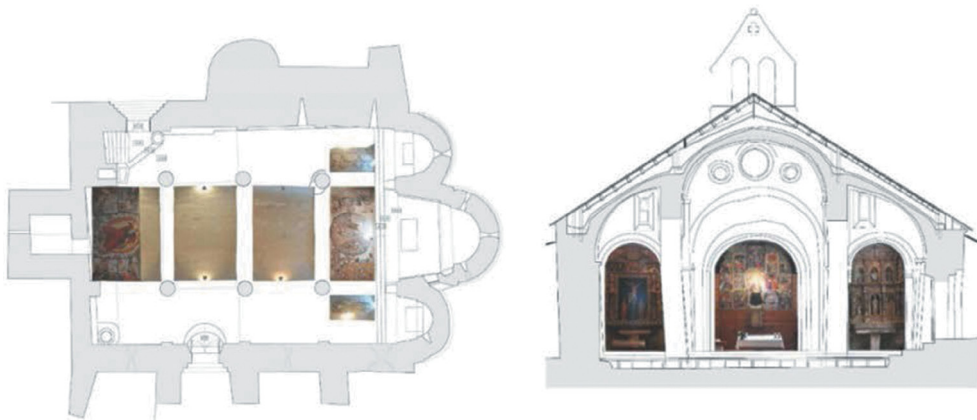


Figure 1: Plan and section of Santa Maria d'Arties [9].

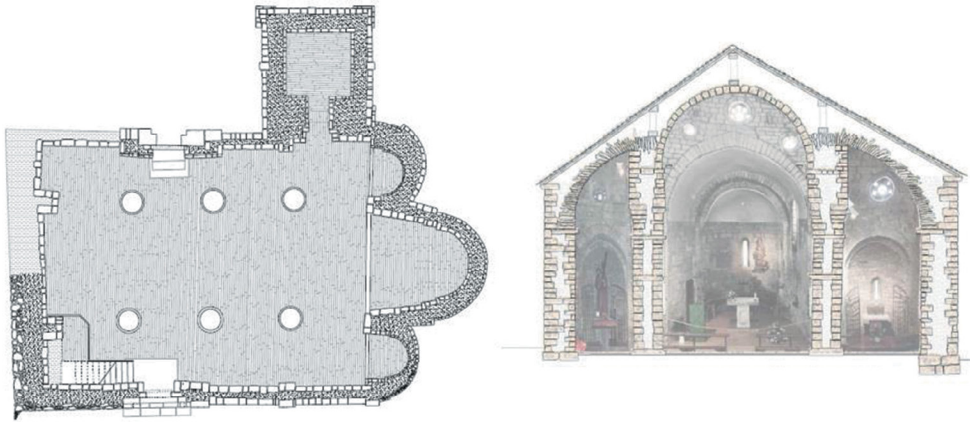


Figure 2: Plan and section of Mair de Diu dera Purificacion of Bossost [10].

church are compared with the maximum deformation. Artes measures 9.76 m in height and 13.86 m in width, and the maximum displacement of the upper part of the walls of the apse measures 0.03 m.

The church of Mair de Diu dera Purificacion of Bossost (Fig. 2) is characterized by a stylistic unity of the 12th century. It was built with a basilica floor plan and three apses. The bell tower is at the northeast facade and the axis of the construction is rotated 15 grades from the typical east–west orientation. The inner space can be defined as a rectangular space of approximately  $19.86 \times 10.97$  m. The church has six circular columns of freestone. Bossost has no relevant deformations detected.

The church of Santa Eulària d'Unha (Fig. 3) is a church with a basilica floor plan with three naves. The apse has a Lombard style. In the northeast, there is a bell tower built in the 18th century. The construction is raised with the typical east–west orientation and the interior measurements are  $21.27 \times 10.92$  m. The displacements of the vertical structure were measured at the base of the walls and columns. Values are expressed in degrees ( $\alpha$ ). The northern wall of Unha is  $2^\circ$  out of plumb, the inner columns  $4^\circ$  and  $3^\circ$ , and finally, the southern wall has plumbed  $4^\circ$ . The horizontal displacement of the upper part of the northern wall measures 0.09 m, and the wall measures 1.13 m in width at the base. If the difference

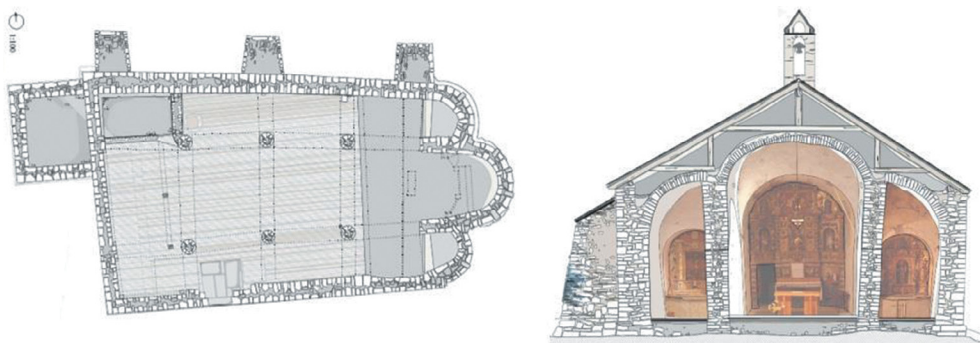


Figure 3: Plan and section of Santa Eulària d'Unha [11].

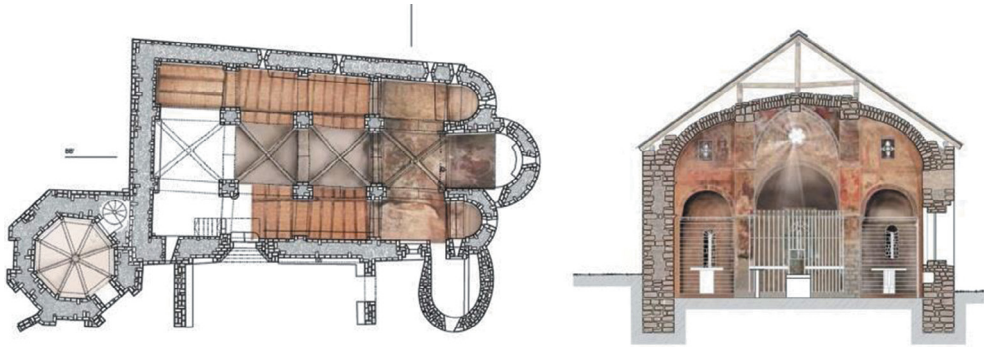


Figure 4: Plan and section of Sant Andreu of Salardú [12].

in height is related with the width of walls and their displacements, Unha is the second most deformed church. There is a difference of 1.45 m in height (measured at the keystone) between the lateral vault and the central vault, and the span of the central vault is 0.11 m larger at the springing than the distance between the supports at the base. The arch of the presbytery of Unha measures 6.66 m in height and the apse has a width of 10.21 m. The maximum displacement of the upper part of the walls of the apse is 0.09 m.

The church of Sant Andreu of Salardú (Fig. 4) belongs to late Romanesque (13th century). The construction is raised with an east–west orientation. The interior measures are  $24.27 \times 11.76$  m. In the southeast angle, there is a bell tower from the 15th century with an octagonal floor plan and Gothic vaults. Inside the style of the church is Gothic because the space is covered with ribbed vaults over cruciform columns. The church has no measured deformations.

The church of Santa María de Vilamós (Fig. 5) has a central nave and one of the lateral naves covered with barrel vaults, with the other lateral nave covered with a half barrel vault. The vaults are supported by four circular columns and two cruciform columns. The orientation of the church has a rotation of  $15^\circ$  from the east–west axis. The inner space can be defined as a rectangular space of approximately  $24.09 \times 9.85$  m. Deformations have been measured in degrees ( $\alpha$ ), taking as reference the bottom part of walls and columns: the northern wall is  $2^\circ$  out of plumb, the inner columns  $1^\circ$  and  $3^\circ$ , and the southern wall  $2^\circ$ . The upper part of the northern wall, which measures 1.66 m in width at the base, has suffered a displacement of 0.011 m. The difference in height of the keystones between the central and

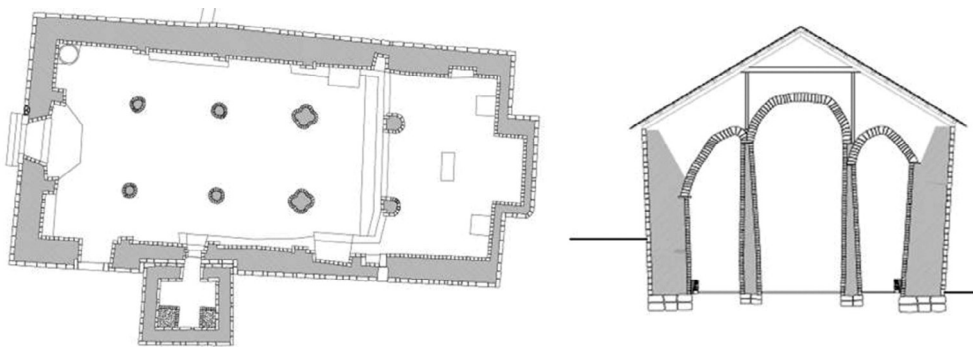


Figure 5: Plan and section of Santa María de Vilamós [13].

lateral vaults is 1.25 m. The northern vault increased its span by 0.34 m, whereas the southern vault increased its span 0.34 m. The church of Vilamos has a height of 6.08 m measured at the central arch of the presbytery, a total width of 9.62 m and maximum displacement of the upper part of the walls of the apse of 0.011 m.

The three main structural criteria are resistance, stiffness and stability [14]. The concept of stability rules masonry constructions. Thus, historical buildings such as the churches of Aran Valley are ruled by this criterion. Those structures have to be assessed considering the overall stability, which is sometimes due to its capacity to assume geometrical deformations.

The theoretical framework for the assessment of masonry structures is currently well developed, according to the principles of limit analysis defined by Heyman [15] and developed by many authors such as Huerta [16] and Block et al. [17]. Those structures are subjected to compression, being far from its mechanical limits, even in the largest buildings. Therefore, they are considered to have infinite resistance to compression. In addition, the tensile strength is considered to be null, and friction prevents sliding between pieces. New methodologies for the assessment of masonry structures based on limit analysis have been developed in recent years. To cite some examples, ref. [18] proposes a methodology that combines integral 3D models and limit analysis structural assessment. Other approximations are based on strategies, such as rigid block limit analysis [19] or the Load Path Method [20] for the assessment of masonry arches. Moreover, refs. [21, 22] developed a 3D funicular analysis, based on limit analysis, for the assessment of masonry vaults. Despite these developments, due to the typology of the structure being analysed, the investigation is based on traditional graphic statics.

#### 4 CASE STUDY: SANTA EULÀRIA D'UNHA

Santa Eulària d'Unha is a Romanesque church built at the end of the 11th century. It is located on top of a rocky hill in the little village of Unha, 9 km from Vielha. Throughout history, it has suffered several transformations, most of them due to the displacement of the masonry. There is very little documental evidence of these changes, but the historical context and the assessment of the masonry make it possible to set a general chronology.

The main access to the building is located in the southern facade, where most of the openings can be found. The northern wall is completely blind, and three large buttresses were built at some time to balance the thrusts of the vaulting. In 1716, the bell tower was built with a total height of 25 m, supported over the west facade of the church. Regarding the roof, the marks on the masonry of almost three different inclinations reveal several changes.

All these modifications have changed the stability conditions of the structure, altering the distribution of loads. These processes, together with the settlement of the masonry through time, have deformed the architectural elements. The measures are not constant, the geometrical forms are irregular and neither pillars nor walls are aligned. Regarding materials, the church is basically built with little stones and lime mortar with low mechanical properties. The roof presents the typical solution with a timber frame and slate tiles, supported over pillars and walls.

#### 5 METHODOLOGY

The assessment of the equilibrium conditions of the structure is based on the widely developed principles of limit analysis, determining possible stable solutions of the structure through lines of thrust. This type of study is especially helpful when visualizing whether the collapse mechanisms of a structure can be activated. The limit analysis enables the characterization of the stability conditions of the current structure. The described strategies are commonly used in the assessment of masonry structures [23–25].



The stability of the structure is assessed by means of two geometrical models: the current shape of the masonry and a theoretical initial shape before deformations occurred. This can be achieved by a drawing restitution process assuming that the supports did not change their position in the ground and were perfectly vertical.

Stability is analysed for both the hypothetical and current shapes of the structure. It evidences the differences between the stability conditions. The most deformed sections obtained from the Total Station survey are analysed for the current state. The specific weight of materials was defined according to usual values. For masonry, it is  $25 \text{ kN/m}^3$  weight, and for roofing, it is  $1.5 \text{ kN/m}^3$ . The supports of the timber structure were placed over the vertical masonry structural elements of the church.

The assessment of the stability of the church is performed for both the initial section and current deformed section. The analysis of the theoretical initial geometry revealed that the stability of the church was already delicate. It is possible to draw a range of possible solutions considering various hypotheses, although they are near the limits of the masonry (Fig. 6). However, according to the principles of limit analysis, if it is possible to find a network of compression forces in equilibrium with the applied loads that fits within the section of the structure, it will be stable and will not collapse. So, if it is possible to find a thrust line within the masonry, the structure will be stable and will not collapse. Moreover, it has to be taken into account that a hinge will appear if the thrust line touches the edge of the structure. An arch with three hinges is stable, but the formation of a fourth hinge will transform the arch into a mechanism and it will collapse [26].

When analysing the current deformed shape, the lines of thrust tended to escape its limits in the base of the pillars. This circumstance leads to the conclusion that an extra load in the upper side of the vaults was dismissed in the hypothesis of the current roofing solution.

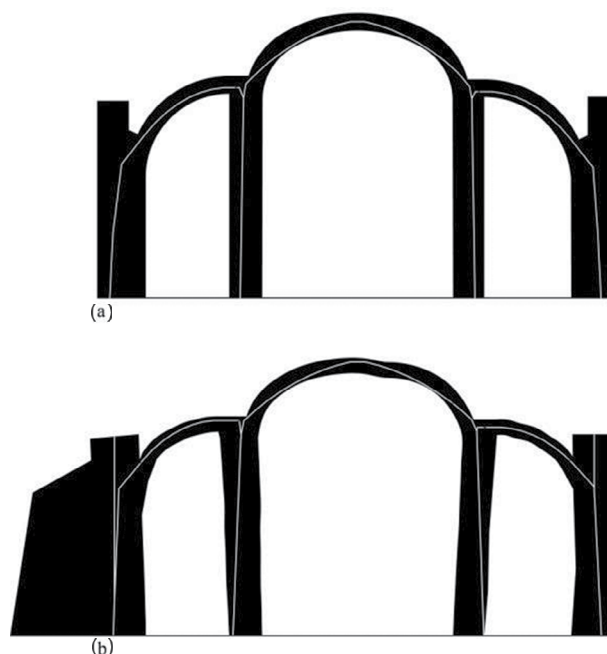


Figure 6: Limit analysis results: (a) theoretical initial shape; (b) current deformed shape.



Increasing the weight of the lateral vaults had a great redirecting effect on the thrust lines (Fig. 6), and it was possible, as in the initial shape, to find a valid range of solutions of thrust lines for the current state of Santa Eulària d'Unha.

Finding thrust lines within the section of the central vault has been particularly challenging because of its lowered geometry. In addition, the slenderness of the pillars was not favourable. The results highlight the importance of the loads applied on the upper side of the lateral half vaults. This set of constructive elements works as a buttressing system for the central lowered vault, counteracting the large horizontal thrusts.

By comparing results of the analysis with the church itself, it can be proven that the zones where the lines of thrust approach the boundaries of the section are traduced into physical cracks.

## 6 CONCLUSIONS

The survey of the five basilica floor plan churches of Aran Valley allowed us to quantify some of the deformations suffered by these masonry structures. Moreover, the typological assessment evidences the wide variety of geometrical solutions that can be found based on the same floor plan typology.

The results obtained in the limit analysis of Santa Eulària d'Unha revealed that, according to the principles of the procedure, the structure should have been stable. This finding highlights the importance of the loads on the extrados of the lateral half vaults in absorbing the considerable horizontal thrust caused by the lowered geometry of the central vault.

The church, which has large deformations, stands out as an example of a masonry building challenging the boundaries of stability. Like most of the churches in the Aran Valley, it evidences the importance of defining an analysis methodology based on suitable procedures to understand the behaviour of the masonry structure.

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