LAYOUT OF THE GOTHIC OCTAGONAL DOME OF TORTOSA CATHEDRAL

J. LLUIS I GINOVART, S. COLL-PLA & A. COSTA-JOVER
Department of Architecture, Rovira i Virgili University, Spain.

ABSTRACT

The Cathedral of Tortosa (1345–1757) is one of the most important Gothic buildings of Catalonia (Spain). Its relevance is partly due to the large number of documents that have been found from the beginning of its construction. An important document is Guarc's parchment (c.1345–1380), which contains the drawing of the plan view of the cathedral. The basilica floor plan, however, with three naves, a dome and a polygonal apse was never built.

The study of the Gothic layout of Guarc through its imprints allows our determining the methodology of the outline of the cathedral. The outline of the octagonal dome is determined by the geometric method of the design. At the same time, the proportional theory that exists behind the parchment allows the transport of the project to the site using simple metrological arithmetic operations.

The methodology used for the study of the document is based on the analysis of its auxiliary layout and the lines and compass points on which the final plan is drawn. An analysis of the preceding imprints to the final plans allows us to establish an interpretive methodology of the possible graphical construction. Later, this documentation will be compared with the built cathedral.

Keywords: dome, medieval drawing, medieval geometry, octagon, Tortosa Cathedral.

1 INTRODUCTION: THE GOTHIC DRAWING AS A COMMUNICATION TOOL BETWEEN THE ARCHITECT AND THE PROMOTER

The ecclesiastical figures of the bishop and Chapter with the medieval architect envisioned the cathedral as the city of God built by men. It is possible to relate the knowledge of ecclesiastical promoters with the medieval masters. A vehicle for attaining this knowledge is the building project, essentially embodied in the drawing of Gothic cathedrals.

This relationship between the ecclesiastical figures (bishop and Chapter) and the medieval architect was emphasized by Wilhelm Worringer (1881–1965) in *Formprobleme der Gotik* (1911) and by Erwin Panofsky (1892–1968) in his work *Gothic Architecture and Scholasticism* (1951). The search for the canons of medieval creation is partly found in the cosmology of the *Timaeus* by Plato (c.429–347 BC), which is recognized by Francis Macdonald Cornford (1874–1943) in *Plato's Cosmology. The Timaeus of Plato* (1937). Thus, one of the points of overlap between clerical and artisan statements is established in the measurement and proportion of architecture. Otherwise, Gothic drawings are considered as a communication tool.

From the perspective of Gothic cathedrals, Otto von Simson (1912–1993), in *The Gothic Cathedral: the Origins of Gothic Architecture and the Medieval Concept of Order* (1956), approached the question of cult resources (treatises and codices of reference) and looked for evidence in the *Civitatis Deis*, *De Ordine* and *the Musica* of Saint Agustín (354–420) (Fig. 1). This research was supplemented by examining authors, such as Boecio (480–524) and his *De consolatione philosophiae* and the *Musica* as well as the main commentators on Plato, such as Calcidius (f. 350) with *Timaeus translatus commentarioque instructus*, Marciano Capella (f. 430) with *De Nuptiis Philologiae et Mercurii Comentarii* and Macrobius (f. 400) with *Somnium Scipionis*.

Thus, the study of the graphic representations of the Gothic world allows understanding the relationship between the medieval architect and his promoter and uses the architectural

Figure 1: Saint Agustín. Cathedral of Tortosa.

drawings as a design element. The architectural drawing is able to transmit the Chapter of the Cathedral the shape of the future construction in the theses established by Roland Recht [1].

The amount of new and existing layouts in the Chapter Archive of Tortosa (ACTo) at the beginning of the construction of the Gothic Cathedral of Tortosa (1345–1441) is exceptional. Especially relevant are the layouts of Antoni Guarc (c.1345–1380) and Basques de Montblanc (c.1345–1377) (Fig. 2), which are considered in the cataloguing of Begoña Alonso Ruiz and Alfonso Jiménez Martín, the oldest existing layouts in Spain (Alonso & Jiménez [2]).

The precedent is the project of François Bucher in *Architector. The Lodge Books and Sketchbooks of Medieval Architects* (1979), a large four-volume work, which ranges from Villard de Honnecourt (Book 1) to Rodrigo Gil de Hontañon (Book 8) and three loose-leaf, dismantled sketchbooks (Books 9, 10 and 11). This project was completed with the catalogue of the exhibition *Les batîsseurs de cathédrales gothiques*, which occurred in Strasbourg in 1989 (edited by Roland Recht); the work of Valerio Ascani, *Il Trecento disegnato: Le basi progettuali dell'architettura gotica in Italy* (1997) and the drawings of Viennese Gothic architecture by Johann Josef Böker, *Architektur der Gotik: Bestandskatalog der weltgrössten Sammlung an gotischen Baurissen im Kupferstichkabinett der Akademie der bildenden Künste Wien* (2005). Finally, *Architectural Layout: Design, Structure, and Construction in Northern Europe* (2006) by Marie-Thérèse Zenner comprises the main catalogue for Gothic design. This collection compiled the Gothic and late Gothic main references made by Begoña Alonso Ruiz and Alfonso Martín Jiménez in *La Traça de la iglesia de Sevilla* (2009) and *El arquitecto del tardogótico a través de sus dibujos* (2011).

Interest in the design of Gothic architecture in Spain has led to specific studies of some of its plans. These plans include the bell tower of the church of St. Felix in Girona (1368) (Chamorro [3]; Chamorro & Zaragozá [4]). Contributions have also been made by the historiography of late Gothic architecture, with the ground plan of Seville Cathedral in the Convent of Bidaurreta (1433?, 1490?) (Jiménez [5]; Alonso & Jiménez [6]).
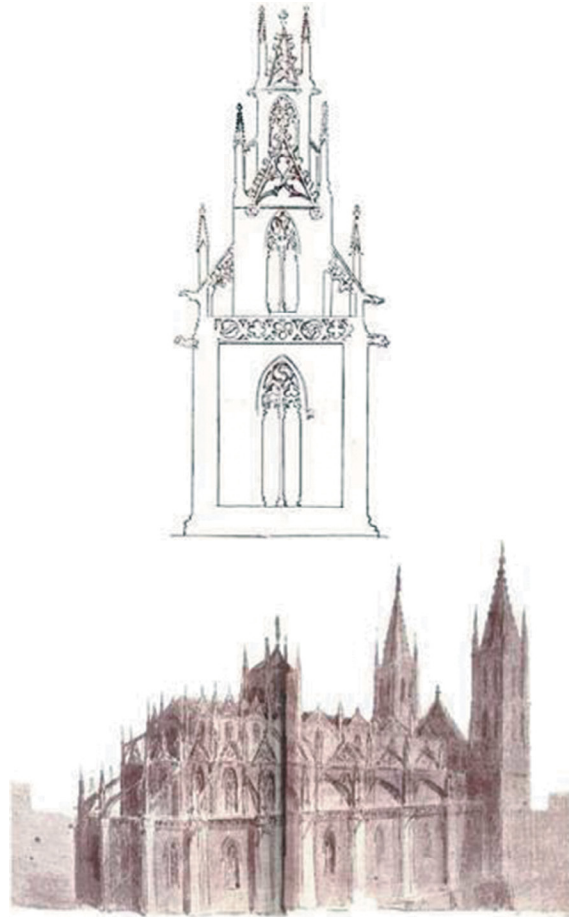
Figure 2: Reproduction and reconstruction of Basques de Montblanc (c.1345–1377) (José Matamoros [10]).

The existence of the drawings coincides with the existence of the sources of Timaeus in the same file as the codices. At the ACTo (Arxiu Capitular Tortosa) nº80 (146r–155v.14), we find the translation of Calcidius' *Timaeus* (f. 350) with part of his *commentary* (f. 155v.15–66) of the 12th century. A fragment of *De Nuptiis Philologiae et Mercurii*, Martianus Capella's Geometry (f. 430) of the VII Book of Geometry and ACTo 80 (f. 160 v.28–f. 161r.5) of the 12th century, also appears. In addition, the *Comentarii in Somnium Scipionis* of ACTo 236 (f. 1–61v), except f. 39, of Macrobio (f. 400) of the 13th century completes his two books. Finally, another part of Calcidius Timaeus's commentary, in ACTo 236 (f. 39) of the 13th century, appears (Lluis i Ginovart [7]). With all of these resources, a relationship of medieval proportions could be established based on Timaeus, with the design of his project and the construction of the Cathedral of Tortosa (Lluis i Ginovart [8]).

This study is the synthesis of numerous investigations that have been conducted regarding medieval geometry in relation to Antoni Guarc's layouts. The analysis contextualizes Guarc's parchment in the European and Spanish Gothic layouts of the 14th to 15th centuries.

In contrast, the parchment can also be framed in the existing graphical representations in the beginning of the Gothic Cathedral of Tortosa (c.1345–1424). From a geometric perspective, the mathematical context, which could have influenced the methods of the octagonal layout from the auxiliary lines of Guarc's layout, has been revised. The octagon is contextualized in the medieval geometry treaties. The final objective of the research supports the initial study of Guarc's parchment trave (c.1345–1380), which was published as 'Gothic construction and the layout of a heptagonal apse. The problem of the heptagon' (Lluis i Ginovart et al. [9]).

2 GUARC'S PARCHMENT

In the Capitular Archive of Tortosa Cathedral (ACTo), a Gothic layout was released by José Matamoros [10]. The parchment (917 × 682 mm) represents the projection of a plant with the notation 'En Antony Guarç'. In the verse '*Fábrica nº49*', 'Mostra d'En Antony Guarç' appears. It is known as Antoni Guarc's layout (c.1345–1380) (Fig. 3).

The Guarc plan was transcribed by José Matamoros [10] and compiled by Pierre Lavedan [11]. The first study of the plan as a representation of the modular structure of the Gothic design was *La traça de la catedral de Tortosa. Els models d'Antoni Guarc i Bernat Dalguaire* by Almuni and Lluis [12]. Other studies concerning the Guarc parchment based on medieval geometry were published in the exhibition catalogues *Fidei Speculum* (Lluis i Ginovart [13]) and *La Llum de les Imatges* (Lluis i Ginovart [14]).

Guarc represents the plan of a design for the chancel and part of the body of the nave of a cathedral. The system for representing the entire building is an orthogonal projection, *ichnographia*, except for the detail of the door of the adjoining chapel, which is shown as a vertical projection. Matamoros believed that the plan of Guarc was part of the preparations for the Gothic building before Bernat Dalguaire and Benet Basques de Montblanc (1345). Almuni dates it to the resumption of the stonework in approximately 1375 with the onomastic appearance of Guarc in the work accounts from 1379 to 1382. Thus, the layout must be dated to the period 1345–1380 (Almuni [15]).
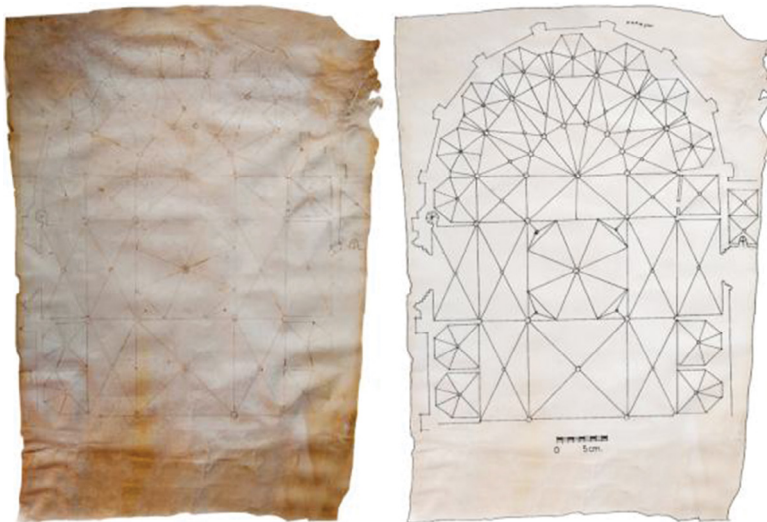

Figure 3: Guarc's parchment, *ACTo. Fábrica nº49* (c.1345–1380).

The basilica floor plan, however, with three naves, a dome and a polygonal apse was never built. The chancel consists of eight interconnected hexagonal chapels with *heptapartite* vaults and a square floor plan with a simple vaulted ceiling, which is laid out as a double ambulatory. The ambulatory has a regular layout, with two rectangular sections and seven trapezoidal sections with a displaced keystone. The presbytery has a central keystone and eleven ribs. The body of the three naves contains two complete structural sections. The first section is roofed with a dome, an octagonal plan and an *octopartite* vault with intermediate ribs; the second is a simple vaulted ceiling. The lateral naves are half the size of the central module and have a rectangular floor plan with an extended vaulted ceiling. The side module has two chapels with a hexagonal floor plan and is roofed by sexpartite vaults.

The floor plan has a modular structure where the basic unit is the chapel. The importance of radial and side chapels as an element of uniqueness originates from the new Gothic liturgical, *Prochiron, vulgo Rationale divinorum officiorum* (1291), which was promoted by the bishop of Mende, Guillaume Durand (1230–1296).

The *Prochiron* defined the correspondence between the *ecclesia materialis* and the *ecclesia spiritualis* (Sebastián [16]). The liturgy characterizes and uses Christian imagery correctly in the liturgical context, which is defined as the *ecclesia materialis* and can be observed by the constructive and theatrical nature of the cathedral space (Van der Ploeg [17]). However, these principles were because of the desire to create an *ecclesia spiritualis* that was represented in the theological and philosophical content of the library of the Chapter Archive. The concept of Durand's new liturgy had replaced an allegorical vision of the liturgy and the practices of the *Gemma animae* (c.1120) by Honorius of Autun (1080–c.1153). In addition, the concept of the *ars*, with the constructive meaning of the *eadifex*, remained anchored in the definitions of the 12th century (Mambelli [18]).

The cathedral, which was conceived as the heavenly Jerusalem, is structured according to the correspondence between the *ecclesias*. The essential elements are the church and its parts, the altar, paintings, curtains and ornaments of the church, the bells, the cemetery, other sacred and religious places and the dedication of the church.

The new design of the apse had already been attempted in Clermont-Ferrand, Narbonne, and was transferred to Catalonia through Girona, where the Chapter required nine chapels in the apse in the contract to build the cathedral in 1312. Another important element is the dome on which the cross is placed; this signifies the perfection and sanctity preached in the Catholic faith and inherited from Saint Peter. Its closest comparison may be Valencia Cathedral (1262), which has a similar structure of a false transept with two lateral doors that culminates with the central dome (Zaragozá [19]). The double structure of the lateral naves is similar to that of the Barcelona Cathedral (1329) (Font [20]).

Guarc's parchment is one of a small number of orthogonal representations of a cathedral project preserved from medieval Europe. There are not many representations of Gothic *ichnographia*. However, some of the representations from existing buildings are conceived as typological notes, such as Villard de Honnecourt (c.1175–1240) (BNP ms.fr.190093), Notre-Dame de Cambrai plans (f. 14v.), the Saint-Etienne de Meaux Cathedral (f. 15r) and Vaucelles abbey (f. 17r) (Lassus [21]). Other representations are considered examples of models, such as those attributed to Michel de Fribourg, which are the Paris and San Croix de Orleans (1388) cathedrals, and the Musée de l'Oeuvre Notre-Dame de Strasboug (Inv.29, Inv.21) (Bucher [22]; Vandekerchove [23]).

Finally, other plans are considered a global project from a part or the totality of the buildings. Therefore, these plans have a mission to advance the future of the buildings, and they relate to Guarc's parchment.

The global projects are as follows: the Steyr plan (Vienna Akademie ABK nº17052), Zagreb (ABK nº16926), Augsburg (ABK nº16846), Kuttemberg (ABK nº16841) (Bucher [22]), the Nürnberg plan (Germanisches National Museum) (Bucher [24]), the Milan cathedral project that is attributed to Henri Parler (c.1392) (Musée de l'Oeuvre Notre-Dame de Estrasburgo, Inv.nº29), the plan from Antonio de Vicenzo (1390–1392) (Museo de San Petronio de Bolognia, cart.389, nº1) and the (S2–S3) Siena Duomo and Museo dell'Opera della Metropolitana plans (Carli [25]; Ascani [26]; Ascani [27]).

Otherwise, partial Gothic masonry projects can be found. This is the case of A Project of Colonia (c.1280–1308) (ABK nº16873) and the E attributed to Master Johanes (c.1310–1320) (Dombauarchiv des Metropolitankapitels Cologne). The project of the presbytery of the Cathedral of Strasbourg (Musée de l'Oeuvre Notre-Dame de Strasbourg, Inv.nº6 v.), the deformation (Inv.nº06/10/11) and façades (Inv.nº15/16) and the draft counter (c.1350) (Hauptstaatsarchiv of Stuttgart Inv No. N201) also include partial Gothic masonry projects. Other examples are the tower plans of Vienna's cathedral (Museen der Stad Wien, Inv.nº105067) (Vrijs 1989 [28], p. 412), the stair plans of Vienna (ABK nº16.855) (Vrijs 1989 [28], p. 372) (ABK nº16953) (Bucher 1979 [29], p. 72), and the examples from Strasbourg's stairs ((ABK nº16.832) and (Victoria and Albert Museum nº3550)) (Recht [30]).

In Spain, the late Gothic drawings with complete floor plans are dated after Guarc's plan. This is the case of the Seville Cathedral, the Convent of Bidaurreta (1433?, 1490?) and the parchment by Bartolomé de Pelayos for the cathedral in Coria (1502) (Sánchez [31]). In addition, there are the floor plans of the Church of San Bartomé in Javea (1513) (Zaragozà [32]), the Church of Segovia (1524) by Juan Gil de Hontañón (Casaseca [33]), the plans of Juan de Alava in the convent of San Esteban in Salamanca (1524) and the Salamanca Cathedral (c.1537) (Alonso & Jiménez [34]).

3 GUARC'S IMPRINTS

The methodology used for the study of the document is based on the analysis of its auxiliary layout and the lines and compass points on which the final plan is drawn. The studies (ms. 1092 Stiftsbibliothek) from Sant Gall (c.820) are a model to explore Guarc's layout (Horn & Born [35]; Fernie [36]; Horn & Born [37]; Nees [38]; Sanderson [39]).

An analysis of the preceding imprints to the final plans allows us to establish an interpretive methodology of the possible graphical construction (Fig. 4). Several points on the support penetrate the parchment. Some points are located on the perimeter to fasten the parchment to its support (Pa1). Other points are from compass marks that could be used to transport measurements (Pa2) or for tracing circumferences, which penetrate and break the surface of the parchment (Pa3).

The lines are drawn using a punch for straight sections (La4) or through a two-pointed compass for circular layouts (La5). Two different types of graphic techniques draw the lines: some are laid out with sharp tools that alter the surface of the skin; others are similar to graphite strokes. The technique that uses sharp tools frames the drawing and establishes the proportions, whereas the graphite strokes were placed as auxiliary strokes and then fixed with final ink strokes (La6).

An analysis of the auxiliary lines and points gives us the drawing sequence and determines the number of graphical operations. Some points are used only once, and others are used twice; some others, such as PO.1, are used up to five times, which comprise most of the base of the auxiliary drawing (Fig. 5).
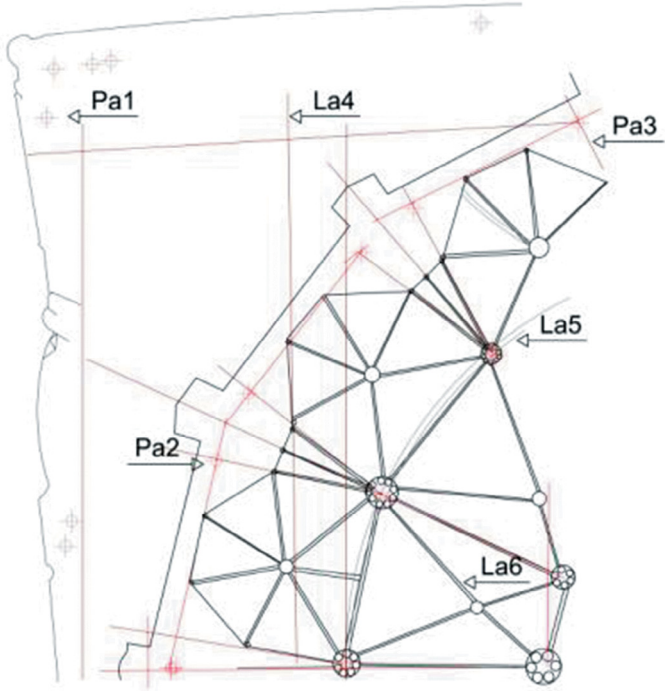
Figure 4: Imprints of points, auxiliary lines and layouts.

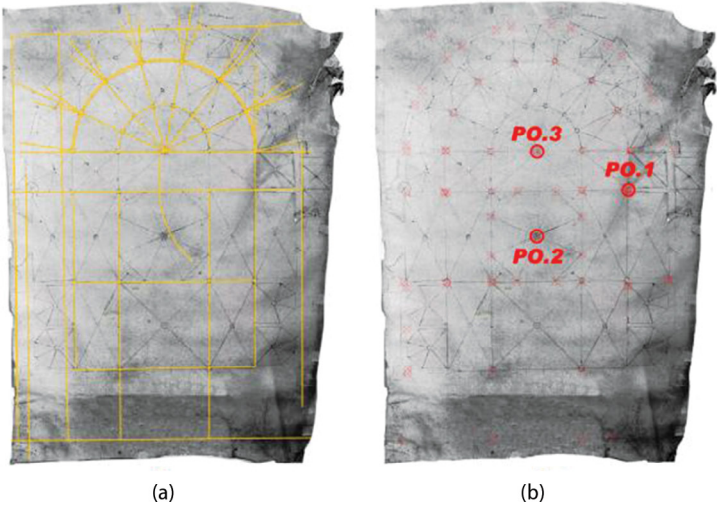(a)   (b)

Figure 5: Auxiliary typology imprints. (a) Lines; (b) points.
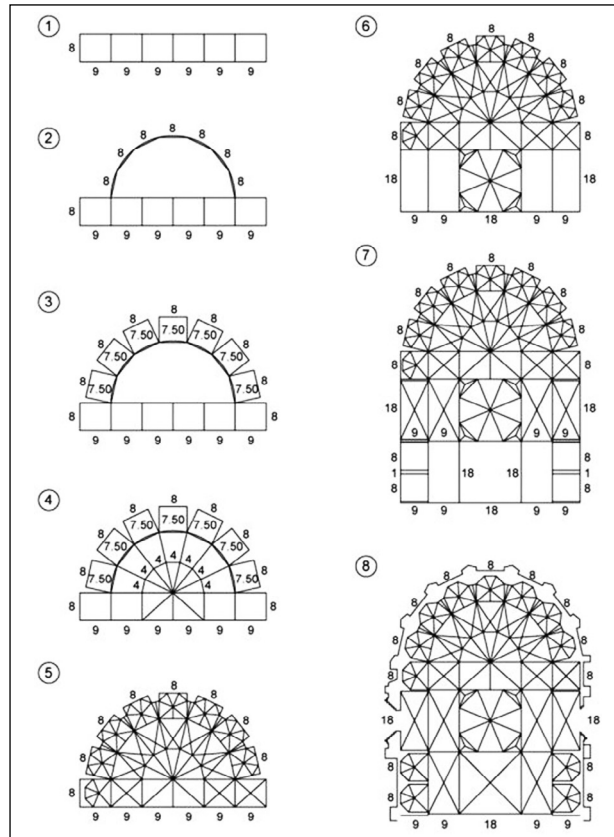
Figure 6: Geometric constructions of Guarc's parchment.

Thus, the following path points are established: PO.1, the origin of many layouts and measurement transport operations; PO.2, the centre of the dome; and PO.3, the choir key and layout apse centre. The dome is based on the main line, T2.1, at the foot of the presbytery. The structural square is obtained from where the dome intersects with P1, P2, P3 and P4 and where a compass point exists.

An initial module makes the lace parchment. Guarc divided the width of the cathedral into six parts (91 mm), a measurement that is considered a unit pattern. The rectangle, next to the primary key of the presbytery, has a ratio of 91/82 mm. The proportion between the side chapel's width and the separation wall is 8/1 and makes the modulus of the side nave (9:18 units) the central one. In conclusion, Guarc uses the 18/8 numerical relationship between the nave and the side chapel or the 9/8 tonal relationship between the ambulatory and the radial chapel (Fig. 6).

4 THE DRAWING GEOMETRY OF THE OCTAGON

The layout of the dome drawn on the parchment requires a method for constructing the octagon. Therefore, Guarc takes the main line, T2.1, at the foot of the presbytery as the base and constructs the structural square, which contains the dome. A compass point is observed at P1, P2, P3 and P4, which was how the centre of the square, PO.2, was laid out. This point is determined by the intersection of the diagonals (P1–P3) and (P2–P4) where the auxiliary layouts of graphite are still visible. The opposite vertices of the square, P1 and P3, have two compass
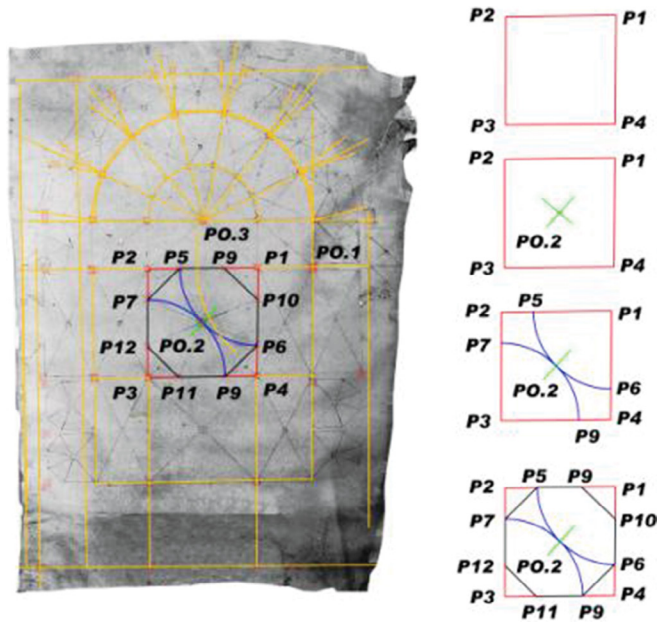
Figure 7: Sequence of the octagonal layout.

marks unlike the rest. The points P5 and P6 are obtained by rotating the segment (P1–PO.2) on the vertex (P1). The same sequence is conducted on point P3 to obtain points P7 and P8. The distance (P5–P7) and (P6–P8) is the measurement of the side of the octagon. Points P9, P10, P11 and P12 are obtained by the reiteration of this measurement with a compass, whose marks can be observed on each point (Fig. 7).

Euclid's *Elements* (c.325–265 BC), which was translated by Adelard of Bath (1075–1166) in approximately 1142, does not address the layout of the octagon based on the inscription of the octagon in the square. The mathematical syntaxes in *Almagest* by Ptolemy (c.85–165), which was translated by Gerard of Cremona (1114–1187) in approximately 1175, also do not address the layout of the octagon.

However, practical methods for the construction of this figure were used in the late classical world. This is the case of the Gromatic text *Fragmentum de hexagono et octogono* (Fig. 8a), which is attributed to Marcus Terentius Varro (116–27 BC) (Bubnov [40]). This text contains the drawing of the octagon, whose squaring construction was widely used in Roman flooring (Watts 1996[41], p. 165–181). This method was also considered by Hero of Alexandria in the *Metrica* (LI.XVIII) (Fig. 8b), 'c.20–62'. This method is considered a reference for the plan by *Horologion des Andronikos* (Tower of the Winds, Athens, s.I aC) (Svenshon [42]) and in some apses with an octagonal layout (Cantor [43]; Özdural [44]). The pseudo-Heronian *De mensuris* also contains a construction of an octagon inscribed in a square (Heiberg [45]; Høyrup [46]).

In the Middle Ages, there was a method originating from the Arab tradition of Abū al-Wafā Al-Būzjānī (940–998) *Kitāb fī mā yaḥtāju al-ṣāni' min al-a'māl al-handasiyya* (*Book on those geometric constructions which are necessary for craftsmen*) 'c.993–1008'. In the method of W79, an octagon is inscribed in a square in the tradition of *De mensuris* (Fig. 8c). This method would have been one of Guarc's direct predecessors. In Western culture, there is the figure of the *mediatrix*, which mediates between the square and circle of *De triangulis* 'c.1250' of Jordanus Nemonarius (1225–1260) (P.IV.15) (Beujouan [47]) (Fig. 8d).
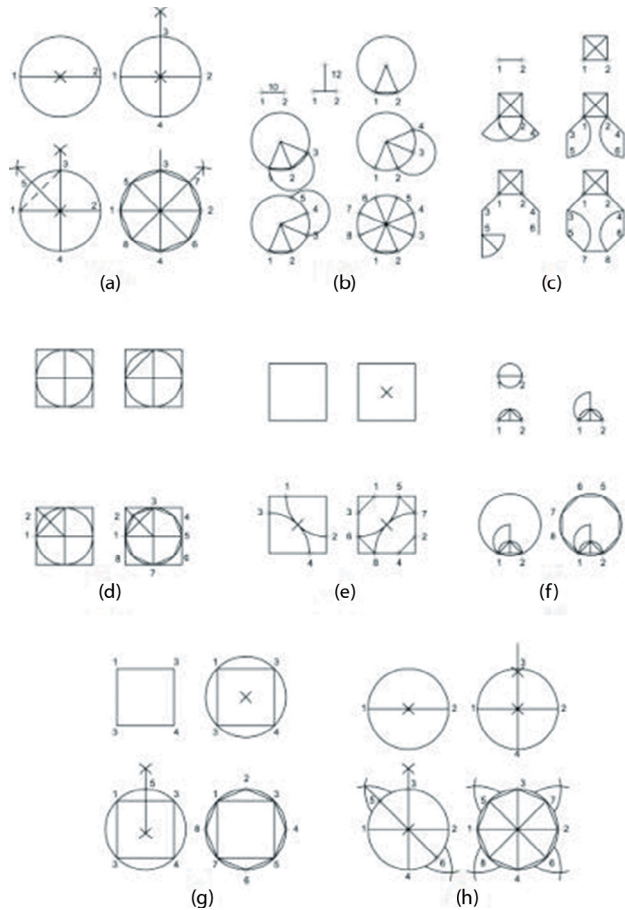
Figure 8: Octagonal layout. (a) Gromatics (c 100 BC); (b) Herón Alexandria (c.60); (c) Abu'l-Wafa (c.990); (d) Jordanus Nemonarius (c.1250); (e) Roriczer (1488); (f) Leonardo da Vinci (1478–1518); (g) Dürer (1525); (h) Arfe (1585).

The Gothic design of the octagon appeared in the *Geometrie Deutsch* (1488) (f. 3r.) by Matthaüs Roriczer (c.1435–1495). The octagon is drawn by inscribing it in a square and translating the measure of the semidiagonal with centre in the edge of the square. (Hoffstadt [48]; Shelby [49]; Recht [50]; Roriczer [51]) (Fig. 8e). An operating system was produced to simulate the construction of WG18 in the Frankfurt album '1560–1572' (Bucher 1979 [29], p. 219). The method is similar to the approach used 100 years earlier by Antoni Guarc (c.1345–1380). It is also similar to W79 by Abū al-Wafā Al-Būzjānī (c.993–1008).

The geometries of the Renaissance also considered the problem of the octagon. Luca Pacioli (1445–1517) considers this in the *Summa de Aritmética, Geometría, Proportioni et Proporcionalità* '1494', where he uses a method of squaring the circle that is similar to Nemonarius (Pacioli [52]). He also addresses the issue in *Divina proportione* (1497) and applies integer arithmetic ratios between the diameter and the side of the octagon (Pacioli [53]). Leonardo da Vinci (1452–1519) in the Windsor codex 12542 (1478–1518) constructs the circle that is inscribed on the side of the square of the isoptic of the side of the octagon (Reynolds 2008 [54], p. 51–76) (Fig. 8f). Albrecht Dürer (1471–1528) in the *Underweysung der Messung*

(1525) uses the reiterated process for the partition of the side of the square with the compass, in the tradition of Nemonarius and Pacioli (Dürer [55]) (Fig. 8g).

In the architectural treatises, such as *Il Primo libro d'architettura* (1545) by Sebastiano Serlio (1475–c.1554), the Roriczer method is used (Serlio 1545 [56], p. 19). In Spain, it appears in the architectural manuscript (c.1545–1562) of Hernán Ruiz el Joven (c.1514–1569) (Navascués [57]). The goldsmith Juan of Arfe (1535–1603) of German origin wrote *De varia commensvracion para la escvltvra y architectura* (1585), the most influential geometric treatise of the Spanish Renaissance, which addressed the octagon (Arfe 1585 [58], f. 7v–8r). He designs the figure by using the circumference and its arcs (LI.T1.C2.13) for which the reference could have been Albrecht Dürer. This influence also applies to the octagon that is inscribed in the square (LI.T1.C2.14), which could be influenced by Augustin Hirschvogel (1503–1553) in *Ein aigentliche vnd grundtliche anweysung in die Geometria. Berg und Neuber* (1543). This method is similar to the approach used by Guarc (Fig. 8h). These methods, which were inherited from the W78 and W79 of Abū al-Wafā Al-Būzjānī (c.993–1008), persisted until Lorenzo de San Nicolás (1593–1579) in *Arte y Vso de Architectvra* (San Nicolás [59]).

5 GOTHIC MODULAR STRUCTURE: TRAÇA VERUS FABRICA

The pattern of *Llibres d'Obra* (ACTo) is the *cana* of 8 spans, and the span of 12 fingers. Tortosa's 'cana' is defined in the IX Book, Rubric 15.5 of the *Consuetudines Dertosae* (1272) (AHCTE, Arxiu Històric Comarcal Terres Ebre, cod. 53, f. 256r), and the text *Costums Generals feutes de la insigne ciutat de Tortosa* (1346) (FBMPM, Fundación Bertomeu March Palma Mallorca, f. 100r). The unification of Tortosa's *cana* to Barcelona's (24-VII-1593) was determined to be 1,858 m.

The head of Tortosa Cathedral has metrological proportions of 150 span width, 100 span depth, with squared geometry radiating chapels that have 21 × 21 spans, and the reorganization of the pillars to three *cana*.

The 9/8 ratio is the same between Guarc's geometric layout and the general stake of the radiating chapels of 24 spans width and is drawn to 54 spans. From the geometric point of view, the structure of Guarc's plan has the same modulation as the built Cathedral. If Guarc's project had been executed, the scale value would have been 3 (Fig. 9).
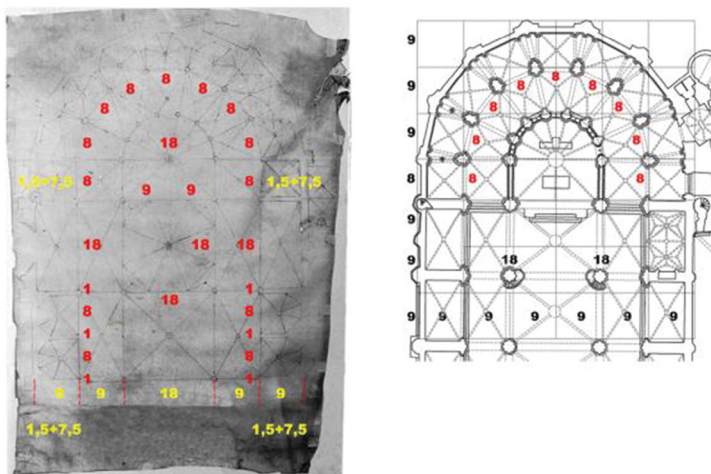

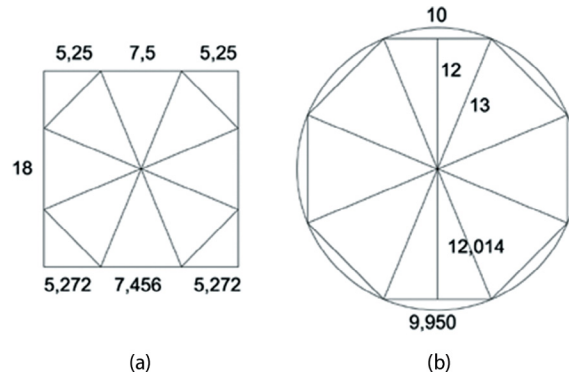Figure 9: Modular structure of Guarc's parchment (c.1345–1380).

Figure 10: Octagonal approaches. (a) Guarç; (b) Herón Alexandria.

Because of the parchment's modular structure, it can be speculated that, with certain comfort, the parchment's measurements can be carried out as an arithmetic relationship. The relation that Heron spread, which was based on the isosceles triangle angle (360/8) (13,10,13), actually (13.9,9.50,13), was known among the Gothic masters. In Guarc's case, the radial chapel floor plan is 8 × 7.5, and the dome is a square layout (18 × 18). The octagonal side is similar to the depth of the radial chapel (7.5), which is obviously an arithmetic approximation (7.456). Heron's measurement errors, similar to Guarc's, are completely negligible in Gothic building measurements (Fig. 10).

6 CONCLUSION

The study has assessed the Gothic layout of Guarc (c.1345–1380) through its imprints, which are the auxiliary layout and the lines and compass points on which the final plan was drawn. It has allowed us to determine the methodology of the outline of the octagonal dome and its geometric method of design. Moreover, the proportional theory that exists behind the parchment allows the transport of the project to the site using simple metrological arithmetic operations.

The geometric layout of the octagon used by Guarc is similar to the layout published in Geometrie Deutsch (1486) by Matthaüs Roriczer (c.1435–1495). The *geometria fabrorum* in Guarc's layout raises two basic issues. On the one hand, the design layout is drawn geometrically with a compass. On the other hand, the masonry transposition has a metrological and arithmetic nature. The parchment structural plan has an arithmetic base of nine and allows us to solve as an algorithm by integer numbers the design layout and its later use in the masonry.

In conclusion, Guarc's plan was never built, but his modular structure is the same as the executed work, from a geometric perspective. If Guarc's project had been executed, the proportional factor would have been 3. Thus, the parchment's measurements can be carried out as an arithmetic relationship. The relationship that Heron of Alexandria spread, which was based on the isosceles triangle angle, was known among the Gothic masters. The radial chapel floor plan is (8 × 7.5), the dome has a square layout (18 × 18) and the octagonal side is similar to the depth of the radial chapel (7.5). Measurement errors of Heron and Guarc are completely negligible in Gothic construction. Thus, the arithmetic operation of Guarc's octagon on a square base 18, whose side is 7.5, the same as the bottom side chapel, has a *Metrica* based on Heron of Alexandria.

Geometrical operations of the octagonal drawing, posed with *circino* (Etym.XIX. XIX.19.10), allude to instrumentally immeasurable metrics. With a changed scale, the design must be moved to the masonry with a different instrument, such as the *linea* (Etym.XIX.18.3) and the *norma* (Etymo.XIX.18.1) which must have a metrological base.

REFERENCES

[1] Recht, R., *Le Dessin d'architecture. Origine et function*, Hazan: Paris, pp. 21–56, 1995.

[2] Alonso, B. & Jiménez, A., *La Traça de la iglesia de Sevilla*, Cabildo Metropolitano: Sevilla, pp. 105–106, 2009.

[3] Chamorro, M.A., Los libros de obra de la iglesia de San Félix de Girona en el siglo XIV. *Actas del Cuarto Congreso Nacional de Historia de la Construcción*, Cádiz, 27–29 enero 2005, S. Huerta, Instituto Juan de Herrera: Madrid, pp. 317–327, 2005.

[4] Chamorro, M.A. & Zaragozá, A., La traza de la torre campanario de la iglesia de San Félix de Gerona, *Goya*, **338(3–15)**, pp. 317–327, 2012.

[5] Jiménez, A., El arquitecto del tardogótico a través de sus dibujos. *La arquitectura tardogótica castellana entre Europa y América*, ed. B. Alons, Silex: Madrid, pp. 63–74, 2011.

[6] Alonso, B. & Jiménez, A., A fifteenth-century plan of the cathedral of Seville. *Architectural History*, **55**, pp. 57–77, 2012.

[7] Lluis i Ginovart, J., Fortuny Anguera, G., Costa Jover, A., de Solà Morales-Serra, P., La sciencia theorica speculativa en la construcción gótica. El promotor eclesiástico de la catedral medieval. *Hispania Sacra*, **LXVI**. Extra I, pp. 7–34, enero-junio 2014.

[8] Lluis i Ginovart, J., La proporción medieval en el diseño y construcción de la catedral de Tortosa. *Archivo Español de Arte*, **LXXXVII(348)**, pp. 321–334, octubre-diciembre 2014. DOI: 10.3989/aearte.2014.20.

[9] Lluis i Ginovart, J., Gothic construction and the traça of a heptagonal apse. The problem of the heptagon. *Nexus Network Journal: Architecture and Mathematics*, **15(2)**, pp. 325–348, 2013. DOI: 10.1007/s00004-013-0152-x.

[10] Matamoros, J., *La catedral de Tortosa. Trabajos monográficos acerca de su construcción y de su contenido artístico y religioso*, Editorial Católica: Tortosa, 1932.

[11] Lavedan, P., *L'architecture Gothique Religieuse, en Catalogne, Valence et Baléares*, Laurens: Paris, 1935.

[12] Lluis i Ginovart, J. & Almuni Balada, V., La Traça de la catedral de Tortosa. Els models d'Antoni Guarc i Bernat Dalguaire, *Lambard: Estudis d'art medieval*, **(9)**, pp. 23–37, 1996.

[13] Lluis i Ginovart, J., Traça de la catedral de Tortosa. *Art litúrgic Diócesi de Tortosa*, Fundació "la Caixa": Barcelona, p. 190, 2000.

[14] Lluis i Ginovart, J., Traza del ábside de la catedral de Tortosa. *La Llum de les Imatges*. Sant Mateu 2005, Generalitat Valenciana: Valencia, pp. 240–241, 2005.

[15] Almuni, V., *La catedral de Tortosa als segles del gòtic*. 2 vols, Fundació Noguera: Barcelona, pp. 362–364, 2007.

[16] Sebastián, S., *Mensaje Simbólico del Arte Medieval. Arquitectura, Liturgia e Iconografía*, Encuentro: Madrid, pp. 352–355, 1994.

[17] Van der Ploeg, K., On the correct usage and characterization of Christian imagery within the liturgical milieu: Durandus of Mende's "Rationale Divinorum Officiorum" and the function of imagery in art. *Speculum a Journal of Medieval Studies*, **78(I.4)**, pp. 1287–1288, 2003.

[18] Mambelli, F., Il problema dell'immagine nei commentari allegorici sulla liturgia: Dalla Gemma Animae di Onorio d'Autun (1120 ca.) al Rationale divinorum officiorum di Durando di Mende (1286–1292), *Studi Medievali*, **45(I.1)**, pp. 121–158, 2004.

[19] Zaragozá, A., *Arquitectura Gótica Valenciana Siglos XIII-XV. Monumentos de la Comunidad Valenciana. Catálogo de Monumentos y Conjuntos declarados e incoados.* Comunidad Valenciana Conselleria de Cultura, Educació i Ciència: Valencia, pp. 65–68, 2000.

[20] Font, A., *La catedral de Barcelona. Ligeras consideraciones sobre su belleza arquitectónica*. Imprenta y Litografía de Henrich y Ca., en comandita: Barcelona, pp. 9–13, 1891.

[21] Lassus J.B.A., *Album de Villard de Honnecourt. Architecte du XIIIe siècle*. Imprimerie imperial: Paris, pp. 112–124, 130–131, 1858.

[22] Bucher, F., Design in Gothic architecture. A preliminary assessment. *Journal of the Society of Architectural Historians*, **XXVII(1)**, pp. 49–71, 1968.

[23] Vandekerchove, C., C32. Plan recto et verso des chœurs des cathédrales de Paris et d'Orláns. *Les bâtisseurs des Cathédrales Gothiques*, Editions les Musées de la Ville de Strasbourg: Strasbourg, pp. 317–318, 1989.

[24] Bucher, F., Medieval architectural design methods, 800-1560, *Gesta*, **11(2)**, pp. 37–45, 1972. DOI: 10.2307/766593.

[25] Carli, E., *Il Duomo di Siena*, Sagep Editice Genova: Genova, pp. 22–24, 1979.

[26] Ascani, V., Le dessin d'architecture médiéval en Italie. En: *Les bâtisseurs du Moyen-Âge. Catalogue, in Les bâtisseurs des Cathédrales Gothiques*, Editions les Musées de la Ville de Strasbourg: Strasbourg, pp. 255–277, 1989.

[27] Ascani, V., Il Trecento disegnato. *Le basi progettuali dell'architettura gotica in Italia*, Viella: Roma, pp. 89–94, 1997.

[28] Vrijs, A. Catalogue. *Les bâtisseurs des Cathédrales Gothiques*, ed. R. Recht, Strasbourg: Editions les Musées de la Ville, 1989.

[29] Bucher, F. Architector. *The Lodge Books and Sketchbooks of Medieval Architects*. Vol. I. New York: Abaris Book, 1979.

[30] Recht, R. (ed.) *Les Bâtisseurs des Cathédrales Gothiques*, Editions les Musées de la Ville de Strasbourg: Strasbourg, pp. 392–408, 1989.

[31] Sánchez, F.M., Martín de Solórzano: la influencia de Santo Tomás de Ávila en los proyectos constructivos de la Catedral de Coria', *Norba-Arte*, **3**, pp. 63–76, 1982.

[32] Zaragozà, A. & Arroyas M., *Una arquitectura gòtica mediterrània*, Generalitat Valenciana: Valencia, pp. 175–176, 2003.

[33] Casaseca, A., Trazas para la catedral de Segovia. *Archivo Español de Arte*, **LI**, pp. 29–51, 1978.

[34] Alonso, B. & Jiménez, A., *La Traça de la iglesia de Sevilla*, Cabildo Metropolitano: Sevilla, pp. 116–117, 2009.

[35] Horn, W. & Born, E., The "dimensional inconsistencies" of the plan of Saint Gall and the problem of the scale of the plan. *The Art Bulletin*, **48(3/4)**, pp. 285–308, 1966. DOI: 10.2307/3048387.

[36] Fernie E., The proportions of the St. Gall plan. *The Art Bulletin*, **60(4)**, pp. 583–589, 1978. DOI: 10.2307/3049839.

[37] Horn, W., & Born, E., *The Plan of St. Gall. A Study of the Architecture & Economy of, & Life in a Paradigmatic Carolingian Monastery,* 3 Vol. University of California: Berkeley, 1979.

[38] Nees, L., The plan of St. Gall and the theory of the program of Carolingian. *Gesta*, **25(1)**, Essays in Honour of Whitney Snow Stoddard, pp. 1–8, 1986. DOI: 10.2307/766891.

[39] Sanderson, W., The plan of St. Gall reconsidered. *Speculum*, **60(3)**, pp. 615–632, 1985. DOI: 10.2307/2848178.

[40] Bubnov, N., *Gerberti postea Silvestri II papae opera mathematica (972–1003)*, Friedländer: Berlin, p. 552, 1899.

[41] Watts, C.M., The square and the roman house: Architecture and decoration at Pompeii and Herculanum. *Nexus Network Journal: Architecture and Mathematics*, ed. Kim Williams. Fucecchio (Firenze): Edizioni dell'Erba, 1996.

[42] Svenshon, H., 'Schlag' nach bei Hheron. Der Turm der Winde im Sspiegel antike Vvermessungslehre'. *Koldewey-Gesellschaft Vereinigung für Baugeschichtliche Fforschung E.V.*

Bericht über die 45. Tagung für Ausgrabungswissenschaft und Bauforschung. Von 30 april bis 4 maig 2008 in Regensburg, Klaus Tragar: Augsburg, pp. 103–112, 2010.

[43] Cantor, M., *Vorlesungen Uber Geschichte Der Mathematik*, V1, Part 2, B.G. Teubner: Leipzig, pp. 377–379, 1907.

[44] Özdural, A., The Church of St. George of the Latins in famagusta: A Case Study on Medieval Metrology and Design Techniques. *Ad Quadratum*, ed. N. Wu, Burlington: Ashgate, pp. 217–242, 2002.

[45] Heiberg, J.L., *Heronis quae feruntur Stereometrica et De mensuris. Heronis Alexandrini Opera quae supersunt Omnia*, Teubner: Leipzig, pp. 206–207, 1914.

[46] Høyrup, J., The rare traces of constructional procedures in practical geometries. *Creating Shapes in Civil and Naval Architecture*, ed. H. & W. Lefèvre: Leiden & Boston, pp. 367–377, 2009.

[47] Beujouan, G. (ed), Réflexions sur les Rapports entre théorie et pratique au Moye Âge., *The Cultural Context of Medieval Learning*, ed. Murdoch & Sylla, Reidel: Dordrecht-Boston, pp. 453–454, 1975.

[48] Hoffstadt, f., *Principes du Style Gothique. Exposés d'après des documents authentiques du moyen Âge avec 40 planches in-folio. A l'usage des artistes et des ouvriers*, A. Franck et Victor Didron: Paris, p. 20, 1847.

[49] Shelby, L.R., *Gothic Design Techniques. The Fifteenth-Century Design Booklets of Mathes Roriczer and Hanns Schumuttermayer*, Southern Illinois University Press: Carbondale, IL, pp. 119–120, 1977.

[50] Recht, R., Le traité de Géométrie de Mathieu Roriczer. Les bâtisseurs du Moyen-Âge. *Histoire et Archéologie*, Dossiers. Novembre: pp. 24–25, 1980.

[51] Roriczer, M., *Matthäus Roriczer, Das Büchlein von der Fialen Gerechtigkeit. Faksimile der Originalausgabe Regensburg 1486, Matthäus und Roriczer, Die Geometria deutsch. Faksimile der Originalausgabe Regensburg um 1487–1488. Mit einem Nachwort und Textübertragung herausgegeben von Ferdinand Geldner*, Guido Pressler: Hurtgenwald, pp. 56–60, 1999.

[52] Pacioli, L., *Summa de Aritmética, Geometría, Proportioni et Proporcionalità*. Paganino de Paganini: Venezia, Dist.4, Cap2, f. 31v, 1494.

[53] Pacioli, L., *Divina proportione: opera a tutti glingegni perspicaci e curiosi necessaria oue ciascun studioso di philosophia: prospettiua pictura sculptura: architectura: musica: e altre mathematice: suavissima: Sotile: e admirabile doctrina consequira: e delecterassi: co'n varie questione de secretissima scientia*, A. Paganius Paganinus: Venetiis, f. 7r-7v, 1509.

[54] Reynolds, M., The octagon in Leonardo's drawings. *Nexus Network Journal*, **10**, pp. 51 –76 2008. DOI: 10.1007/ S00004-007-0056-8.

[55] Dürer, A., *Underweysung der Messung, mit dem Zirckel und Richtscheyt: in Linien Ebnen vo gantzen Corporen*, Hieronymus Andreae: Nüremberg, L.II.14, 1525.

[56] Serlio, S., *Il Primo libro d'architettura di Sebastiano Serlio, bolognese. Le premier libre d'Architecture de Sebastiano Serlio, Bolognoi, mis en lange Francoyse par Iehan Martin*, París, 1545.

[57] Navascués, P., *El libro de arquitectura de Hernán Ruiz, el Joven*, E.T.S.A.M.: Madrid, Lam. XXIII, 1974.

[58] Arfe, J., *Ioan de Arphe y Villafañe. De varia commensuracion para la escultura y architectura*. (Sevilla, 1585), f. 7v-8r.

[59] San Nicolás, L., *Arte y Uso de Architectura*, dirigida al Smo. Patriarca S. Ioseph. Madrid, 1639.