

NATURAL FLOW SYSTEMS: ACQUIRING THEIR CONSTRUCTAL MORPHOLOGY

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ABSTRACT

The generation of flow configuration (shape, structure, patterns, rhythms) is a phenomenon that occurs across the board in natural flow systems. Scientists have struggled to understand the origins of this phenomenon. What determines the configuration of natural flow systems? Are they following the rule of any law? This paper contributes to stress the importance of common features underlying the generation of configuration in animate and inanimate flow systems. More specifically, as a central, unifying principle, the constructal law is introduced and illustrated. Based on studies produced in the light of the constructal theory, we aim to show the oneness of configuration generation in animate and inanimate flow systems. The paper also traces the application of constructal law as a principle of maximisation of flow access in animate morphing configurations, and discusses the need of communication to a specific pattern formation. We further develop the view presented in the paper ‘The constructal unification of biological and geophysical design’ by Bejan and Marden (2009) (*Phys. Life Rev.*, **6**, pp. 85–102).

Keywords: animate systems, communication, constraints, constructal theory, inanimate systems, maximum performance, morphing configuration, unifying law.

1 THE CONSTRUCTAL LAW OF ADRIAN BEJAN

All existing things have a configuration. Is configuration a characteristic of systems? The answer is provided by the constructal law of the generation of flow configuration. First conceived by Adrian Bejan and published in 1996 [1, 2], this law arises from the basic principle that when purpose exists and the flow system has the freedom to morph under constraints, it evolves in time to balance and minimise the imperfections (resistances) toward configurations that flow more easily. The constructal law can be captured in the following statement ‘For a flow system to persist in time (to survive) it must evolve in such a way that it provides easier and easier access to the currents that flow through it’ [2]. The generation of flow configuration belongs in thermodynamics [2, 3]. However, the constructal law is distinct from the other laws of thermodynamics because is concerned with the configuration of nonequilibrium systems. It represents a new extension of thermodynamics – the thermodynamics of non-equilibrium (flow) systems with configuration [2–5].

Bejan and Lorente [2–5] also proposed the methodology to accomplish practical situations – the constructal method. In the light of constructal law and based on the global purpose and on the global constraints, the maximisation of performance of the flow structure is achieved. The optimal structure is constructed by optimising volume or area shape at different length scales, from the smallest constructs (assemblies, building block) up to the larger constructs.

Over the last decade the constructal law has been successfully applied to different topics from the configuration of aerosol agglomeration [6] and liquid droplet impact [7] to animal and plant design [2, 8–12].

2 CONFIGURATION OF INANIMATE NATURAL FLOW SYSTEMS

The phenomenon of generation of flow configuration is ubiquitously in inanimate systems, both in small and large scale systems. Nano- and micron-particle agglomerates often have dendritic shapes instead of spherical shapes. Why does it occur? Reis et al. [6] relied on the constructal law in order

to construct a theory of aerosol agglomeration. Based on the idea that there are not electrically neutral surfaces in contact with air, it is assumed that the forces that make aerosol particles stick onto previously deposited particles are of the electrical type. There are two possible configurations of agglomeration: spherical and conically. The volume of spherical agglomerates is $\sim K^2 t^2$ while the volume of agglomerates of particles with the conical shape is $\sim q_{el}^{1/2} K^{7/3} \mu_{el}^{-1/2} t^{7/3}$, where μ_{el} is the dipole moment, q_{el} is the charge and K is a parameter that depends on the particle size, dipole moment, electric charge, Cunningham correction factor, electric permittivity of the air, surface density of charge and air viscosity.

The constructal law accounts for the time arrow of the physics phenomenon: configurations morph toward easier flowing configurations. So, the constructal idea is that the configuration of the aggregate of particles evolves in time in such a way that the global rate of accumulation of the particles is maximised (i.e. maximum performance). The temporal evolution of the agglomerates volume is presented in Fig. 1. This plot shows that at the critical time, $t_{crit} \sim \mu_{el}^{3/2} K^{-1} q_{el}^{-3/2}$ the volume of the conical agglomerate starts to overtake the volume of the spherical agglomerate. The agglomerate must first grow as a sphere, but then is replaced by a conical (tree-shaped)-configuration because it flows more easily. For water nucleating in ambient air, the critical time for switching between spherical and conical growth as a preferential mode of agglomeration correspond to a diameter of the agglomerate of $\sim 10^{-7}$ m and a critical time of ~ 1 s [6].

The ratio between diameter, d , and length, L , scales of the conical agglomerate $(d/L)_{aggl}$, is estimated to be $\sim (\mu_{el}/q_{el})^{1/3} L^{-1/3}$. Dendritic snow crystals have a length scale of 10^{-3} m ($\mu_{el}/q_{el} \sim 10^{-6}$ m) and $(d/L)_{aggl}$ yields $\sim 10^{-2}$ which agrees with the order of magnitude of the diameter/length ratio of snowflake needles. Another important conclusion presented in this study [6] is that the critical time for switching from spherical to needle-shaped growth occurs when the sphere diameter reaches six particle diameters (i.e. the agglomerate has $(\pi 6^3/6) \sim 113$ particles).

Liquid droplets impact on a solid surface may presents a disc configuration or it develops needles that grow radially. Based on constructal theory, Bejan and Gobin [7] gave a comprehensive explanation of configuration generation and also present a dimensionless number that governs the selection of geometry. This number is defined by the ratio of two lengths, the final radius of the disc that dies viscously, divided by the radius of the still inviscid ring that just wrinkles. Is the constructal theory also able to predict large inanimate systems?

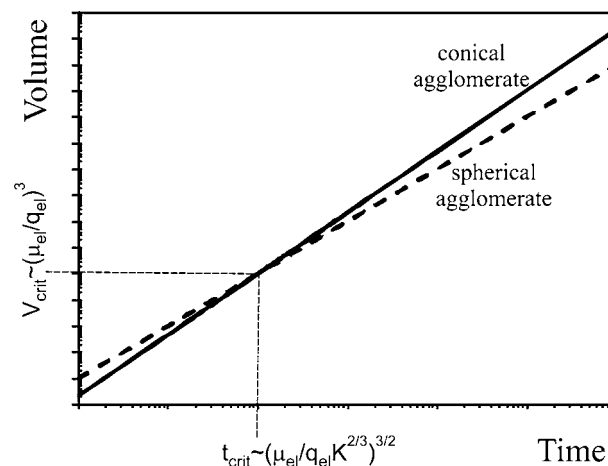


Figure 1: Time evolution of the volume of agglomerates of aerosol (adapted from [6]).

A river basin is the portion of land drained by a river and its tributaries and can cover vast areas (e.g. Yellow River basin in China covers an area of about 742,000 km²). A river basin is an example of an area-to-point flow. Based on constructal law, Bejan [2] has addressed this problem and optimised the channel network that minimises the overall resistance (imperfection) to flow. The purpose of river basins is to collect superficial water and deliver it into the oceans. There are two flow mechanisms to accomplish this purpose: streams (low-resistivity flow) and diffusion or seepage (high-resistivity flow). Rain falls uniformly on the elemental area with a mass flow rate. There is an optimal elemental shape defined as the ratio between the length and width such that the total flow rate collected on the elemental area flows with least global flow resistance from the area through one point on its periphery. The optimised area element becomes a building block with which larger rain plains can be covered. The tree-shaped flow architecture of the river basin represents the best allocation of resistances (optimal distribution of imperfection), and therefore the configuration of the system that allows best flow access from the area to the outlet. This configuration does not constitute a singular result. Bejan and Lorente [2–5] showed that if the purpose is to connect one point (source or sink) with an infinity of points (volume, area, and line), and the fluid has at least two regimes (slow and fast), the configuration that emerges is a tree.

River basins have geometrical features which can be measured, namely the area (measured on the vertical projection), the elongation ratio (the diameter of a circle with the same area as the drainage basin, divided by the basin maximum length), the relief (the difference of elevation between the highest and the lowest points of the drainage area) and the relief ratio (the basin relief divided by the maximum length of the basin). Scaling with size typically follows a simple power law. Several of these allometric scaling laws have been proposed by different authors [13]. The well known Horton, Melton and Hack allometric laws were anticipated based on constructal law as a result of minimisation of the overall resistance to flow [13].

Reis and Gama [14] relied also on the constructal law to address beachface adjustment as a response to wave swash forcing. They showed that beachface slope varies with wave height raised to the power 3/4, and sand grain size raised to the power 4/3. The largest flow system on earth (atmosphere circulation) was also studied from the point of view of the constructal theory [15, 16]. The sun–earth–universe assembly was viewed as a power plant, the power output of which is used to force the atmosphere and hydrosphere to flow. The constructal approach delivered the latitude of the boundary between the Hadley and the Ferrel cells, the boundary between the Ferrel and the Polar cells, the average temperature of the Earth surface, the convective conductance in the horizontal direction, as well as other parameters defining the circulation and the Earth surface temperature.

These and many other examples show that the constructal theory explains much of configuration of inanimate flow systems around us. The cracks patterns evolution during shrinking in soil [2, 5], the dendritic crystals formed during rapid solidification [2, 4], the turbulent flow structure [2, 4], Rayleigh–Bénard convection [2] may also be mentioned here among other examples.

3 SHAPE AND STRUCTURE OF ANIMATE FLOW SYSTEMS

Animate systems are probably the most complex and diverse system in the universe. The so-called ‘life’ covers more than 27 orders of magnitude in mass from molecular and intracellular levels up to whales and sequoias [17]. To understand the underlying morphology behind animate systems, three different approaches can be envisaged: (i) the animate systems as made up of interacting subsystems that provide key functional characteristic of the overall system (e.g. subsystems such as the circulatory system, lungs, kidneys, etc. provide a function that is essential to sustain the entire system); (ii) the colonies of living organisms composed by several individual organisms of the same species living all time closely together for mutual benefit (e.g. stony corals and bacterial colonies), and (iii)

living organisms that are temporally together as an ongoing group to collectively accomplish a particular purpose (e.g. pedestrians accessing shopping facilities or a stadium). Systems at these levels of organisation demonstrate the complementary nature of configuration and function. Therefore, the configuration of animate systems is not only a matter of molecular biology but also of geometry and physics. To make our point, we review some studies based on constructal theory related with the views presented in (i)–(iii).

3.1 Living systems and their components

An animate system is made up of interacting subsystems and each subsystem provides a key functional characteristic of the overall system. Are their configurations providing easier access to the currents that flow through them?

All vertebrates need oxygen for aerobic energy supply. Lungs are the organs specialised for oxygen and carbon dioxide exchange between air and blood. What are the possible configurations for lungs? If the aim is the supply of O_2 to the blood and the drainage of CO_2 from it, there are two flow mechanisms to accomplish this objective: streams and diffusion. Therefore, the lung could be a ducts system or a simple single sac (volume) open to the external air from which the O_2 and CO_2 diffuses between the air and the blood. The access time for a gas concentration to travel by diffusion and by 'streams' through a characteristic length L is $\sim L^2/D$ and $\sim L/u$, respectively, where D is the diffusion coefficient and u the gas speed. Consequently, the possibility of a simple single sac is clearly non-competitive as compared to a ducts system: the former has an access time for streams flow of ~ 1 s (characteristics length ~ 0.5 m and gas speed ~ 0.5 m/s) whereas the latter has an access time of $\sim 10^4$ s (diffusion coefficient $\sim 10^{-5}$ m²/s). Notice that both solutions have internal imperfections (resistances). A duct system has a large friction resistance to airflow whereas the single sac has a large spreading resistance. Why are lungs tree-like structures?

Lungs stem from the need to connect one point (source or sink) with the infinity of points (volume). Constructal theory begins with the objective and constraints. The purpose is to provide maximum access to flow, and the constraint is the thorax volume. The answer is provided by Bejan [2] based on the constructal theory: as mentioned in the last section, if the aim is to connect one point with an infinity of points and there are different flow mechanisms (streams and diffusion) to accomplish the purpose, the architecture that emerges is a tree (Fig. 2). But what are the tree characteristics that provide the easiest way to supply O_2 to the blood and the drainage of CO_2 from it?

Dichotomy (pairing or bifurcation) is an optimised result in tree-flow structures provided by the constructal theory [2–5, 18], as well as the relationship between successive duct diameters or between

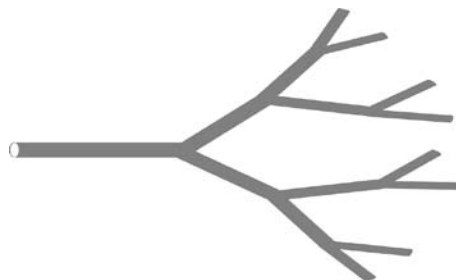


Figure 2: Constructal tree-shaped flow configuration in nature is not toward compactness or complexity, but from the decision to connect one point (source or sink) with an infinity of points (volume, area, and line) maximising the flow access.

successive duct lengths for both laminar and turbulent flows [18]. The relationship known in physiology as Hess-Murray law – the cube of the diameter of a parent duct should equal the sum of the cubes of the diameters of the daughter ducts – is one of the results obtained, and represents the configuration that minimise the global flow resistance for laminar flow subject to the constraints of the space occupied by ducts (Fig. 3). A similar relationship between successive duct lengths is also obtained (i.e. for laminar flow the ratio between the lengths of daughter and parent ducts is $2^{1/3}$). Therefore, the geometric ratio between diameter and length is preserved in going from each duct to its branch in laminar flow, i.e. each duct is geometrically similar to its tributary or collector (for turbulent flow is the ratio between the diameter and the cube of the length that is preserved in going from each duct to its branch) [18].

The respiratory tree was also accessed via the constructal law by Reis et al. [19], with the bifurcations number being the result of this assessment. The flow through the ducts was assumed to be Hagen–Poiseuille and the bifurcating ducts obeying to the Hess-Murray law. The global resistance to flow after minimisation yields the optimal number of bifurcations, N_{opt} ,

$$N_{opt} = 2.164 \ln \left[\frac{0.000235 D_0^4 R_{air} T_{air}}{v_{air} D_g L_0^2} \left(\frac{\phi_{g,0}}{\phi_g} - 1 \right) \right]$$

where v_{air} and ρ_{air} are the kinematic viscosity and density of the air, respectively, L_0 is the trachea length, D_0 is the trachea diameter, D_g is the diffusivity of the gas (e.g. oxygen or carbon dioxide) in the air, R_{air} is the air constant, T_{air} is the temperature, and ϕ_g and $\phi_{g,0}$ are the relative concentration of gas in the alveoli and in the outside air, respectively. Based on data available in the literature for L_0 , D_0 and ϕ_g , the optimal number of bifurcations is obtained by assuming a body temperature of 36°C and taking all pertinent values at this temperature. Using these values in the previous formula, N_{opt} is 23.4 and 23.2 for O_2 and CO_2 transport, respectively. As the number of bifurcations must be an integer, the optimal number of bifurcations of the respiratory tree must be 23. This result is an old result, very well-known in pulmonary physiology.

As all the living beings are subjected to the same average environmental parameters, another important conclusion from N_{opt} formula is that the morphological length D_0^2/L_0 (i.e. the ratio of the square of trachea diameter to its length) must also be a characteristic for humans. This new theoretical result anticipated by constructal theory needs confirmation by the anatomists.

Subsystems such as the kidneys, vascularised tissues and the nervous system are other examples of optimised configurations that have been treated from the point of view of constructal theory [2, 5, 20, 21]. Is the entire animate system also ‘optimised’?

Metabolism refers to the physical and chemical processes that occur in living beings to maintain life. The speed of metabolism is called metabolic rate. In the 1940s, Max Kleiber [20, 22] based on

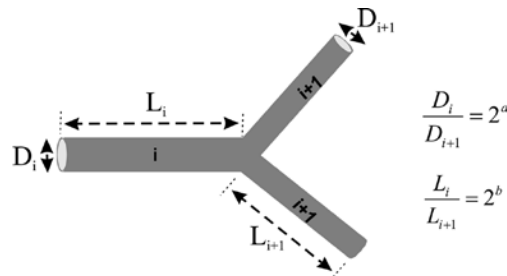


Figure 3: The size of daughter and parent ducts in a constructal fluid tree (laminar flow: $a = b = 1/3$; turbulent flow: $a = 3/7$, $b = 1/7$).

13 mammals, ranging from rats to steers, established for the first time that the correlation of metabolic rate scales as $3/4$ power of the body mass (Kleiber's law). Metabolism is essential for life not only for these 13 mammals. Observations [22] show that this $3/4$ power law for metabolic rates are characteristic of all organisms ranging from the smallest microbes ($\sim 10^{-16}$ kg) to the largest birds, mammals and plants ($\sim 10^5$ kg). A direct explanation for this scaling law was presented by Bejan [2, 23] based on the constructal theory. Kleiber's law was derived by combining a tree-shaped network optimised for minimum pumping power and the convective heat transfer characteristics of two identical fluid trees superimposed in counterflow [2, 23]. The idea that the ubiquity of the $3/4$ power law relies in tree networks common to all organisms is also supported by West and Brown [22].

In summary, metabolic rate is essential and limits almost all biological processes in microbes but also in mammals or birds. The existence of a common $3/4$ power law implies universality that connotes the fact that quite disparate systems behave in a remarkably similar fashion. Therefore, all of them are built from the same 'basic blocks' under the existence of the same powerful constraints at every level of biological organisation (e.g. optimised space-filling, tree networks) because they have survived the process of natural selection [3].

Scaling laws also capture many other essential features of animate systems. Constructal theory is also able to anticipate some of these allometric laws of animals and plants designs (for example, the scaling relationship between the breathing or heart beating time and body mass [24], between the hair strand diameter and the animal body length scale [25], between the flying speed and the body mass [26], and between tree length and the wood mass [27]). The universality and similitude of these allometric scaling laws seems to lay down in the existence of a principle of configuration generation (constructal law) that is underlying the organisation of animate systems.

Flow configuration is generated in space but also in time. Bejan [2] and Bejan and Marden [26, 28] showed that the modes of locomotion (flying, running, swimming) are examples that illustrate the generation of configuration in time and may be also attributed to the constructal principle of configuration generation for maximisation of the flow access in time. Recently, Charles and Bejan [29] collected the heights and weights of the fastest swimmers (100 m-freestyle) and sprinters (100 m-dash) for world record winners, since the beginning of the twentieth century. They then plotted the speed data (based on winning times) versus the size of these athletes. A very interesting conclusion was that the speed records will continue to be dominated by heavier and taller athletes, but this feature is imputable to the scaling rules of animal locomotion [8] 'not to the contemporary increase in the average body size of humans' [29].

In summary, the configuration of living beings (i.e. the subsystems of the living being but also the entire being) has evolved in direction of providing easier access to the currents that flow through it. Another important finding provided by the constructal view of the Kleiber's law [23], as well as the others examples mentioned above [19, 20], is that the optimised space-filling and constructal tree-shaped networks act as basic supportive flow paths along which 'order' need to survive is propagated.

3.2 Colonies of living organisms

A colony refers to several individuals of the same species living closely together, usually for mutual benefit (e.g. formation of a colony is an adaptive behaviour that helps to escape from the attack of predators, to enhance the ability to locate nutrients, etc.). The formation of dissimilar configurations inside similar colonies is especially intriguing. Several authors noticed that for a given set of growth conditions, the colonies experience similar configurations that can be reproduced from the experimental point of view [30–32]. Bacterial colonies that cope with hostile environmental conditions

(i.e. low level of nutrients or hard agar surface) develop branched configuration but colonies that cope with high levels of nutrients develop a compact shape. Stony corals collected from exposed growth sites, where higher water currents are found, present more spherical and compact shape than corals of the same species growing in sheltered sites, which display thin-branched configurations. Therefore, the universality of both configurations suggests that there is an order underlying configurations formation. An adequate explanation requires an understanding of the following: How colony development displays a precise control of the configuration? Why colonies develop characteristic patterns?

Social complexity is a broadly used term that encompasses many colony-level traits and characteristics such as colony size and foraging strategy. Increasing social complexity is associated with an increased use of communication signals to organise cooperative behaviour [33–35]. These signals do not elicit specific responses in themselves, but rather operate in a general manner to alter the probability that individuals will respond to other stimuli. For example, pheromone communication is an effective means of coordinating the activities of insect colonies such as ants, including food gathering, alarm and defence, and even reproduction [35]. The growth of the coral colony (or bacteria colony) also requires communication (i.e. sharing of information) – between cells to create the individual polyp or bacterium, and between polyps or bacteria to create the colony. Communication signals (chemical signals, electrical impulses, vibration signals or other) are necessary for a specific pattern formation. In summary, the interplay between ‘micro’ level (individual polyp or bacterium) and macro level (the colony) play a major role in developing the observed morphologies. This answers our first question but the following question remains: What is the reason for choosing a particular configuration?

The answer is delivered by the constructal theory in [10, 36]. The survival of flow systems calls for configurations that promote an easy flow access. The entire internal and external flow configuration of the systems must contribute to ‘provide easier and easier access to the currents’ [2]. The last section shows that the individual configuration of the living being has evolved in direction of providing easier access to the currents that flow through it. What about flow configuration of the colonies?

Authors [30–32] noticed that the level of nutrients seems to influence the colony configuration. Therefore, the constructal idea is that the preferred configuration is the one that allows the colony to deplete the nutrients as fast as possible. Consider that ϕ is characteristic external length scale of the system, and ζ is the length scale (width) of the branch ($\zeta \ll \phi$). The volumes of branched and spherical configuration scale as $\sim \zeta^2 \phi$ and $\sim \phi^3$, respectively. Because $\zeta^2 \phi \ll \phi^3$, spherical configurations are more effective for filling the space (i.e. to promote an easy flow access in order to extract the available nutrients). In this sense, spherical means perfection, but in reality branched configurations are also likely to occur. How can one reconcile such a contradiction?

Consider, for example, stony corals growing at a rate, v_c , of a few cm/year (e.g. *Pocillopora damicornis* has a growth speed of 1–6 cm/year [36]). If corals grow in open sites, where convective currents are significant, the water speed that transports the nutrients is much higher than the growth speed of the coral. In other words, the convective time scale of water with nutrients is much smaller than the grow time scale of the stony coral. So, the coral always grows inside a region where nutrients are readily available and develops into a round massive configuration because this is the ‘perfect’ configuration (i.e. promotes an easy flow access to extract the available nutrients).

If corals grow in a sheltered site, where water currents are practically absent, diffusion is the most important nutrient transport mechanism. The arrow of time is aligned as following: the living system starts to grow at birth ($t = 0$), and immediately after, nutrients close to the system are quickly consumed and depleted. The concentration of nutrients decreases, which triggers a wave of nutrients

defined by the characteristic length scale, $L_n \sim (D_n t)^{1/2}$ and the speed of propagation, $v_n \sim (D_n/t)^{1/2}$, where D_n is the diffusion coefficient for nutrients and t is the time. The initial speed of propagation of nutrients is greater than any growth speed of the coral, but decreases with the inverse of the square root of time. Thus, v_n drops below v_c when $t > D_n/v_c^2$. Before $t \leq D_n/v_c^2$, the round (massive) shape is the most effective configuration for filling the flow space. But after $t_{crit} \sim D_n/v_c^2$, the living system begins to grow outside the nutrient diffusion region (Fig. 4). To survive, branches (bio-streams) are developed because are low-resistance paths for nutrient access, in a phenomenon that resembles the respiratory tree (section 3.1). This channelling enables the system to continue experiencing growth inside the nutrient rich region from t_{1crit} to t_{2crit} . At times slightly greater than t_{2crit} the coral sticks out of the nutrient region. New branches grow forward in order to access the nutrient region until a new critical time is reached. Each branch generates a new group of branches, and the global feature of this scenario is a tree configuration that represents the most competitive configuration under these circumstances.

A similar explanation can be applied to bacterial colonies. As mentioned before the nutrient content and the agar hardness are parameters that control the colony configuration [30, 31]. Why? According to the constructal law [2, 36] the survival of colony calls for configurations that maximise the colony performance by maximising the global access to nutrients. If the agar surface is soft and nutrient abundant, the colony develops a round massive configuration because, as showed before, is the more effective configuration for depleting the available nutrients. In a hard agar surface (resistance to the movement of nutrient is high) and in a low-nutrient environment, nutrients close to the colony are quickly consumed and depleted. As the speed of propagation of the 'wave' of nutrient is lower than the growth speed of the colony, the development of a round massive configuration is not a good option because the colony starts to grow outside the nutrient region. Branches (bio-streams) are then developed in order to generate low-resistance paths for nutrient access. As for corals, the configuration that emerges is a tree because represents the optimal shape for survival.

In summary, corals and bacteria colonies evolve in time not toward compactness or complexity but toward configurations that make easier for currents to flow.

3.3 Living organisms that are temporally together as an ongoing group

The formation of dissimilar configurations inside a similar group is not an exclusive of coral and bacterial colonies. Observations performed by different authors [17] show that in crowded spaces the movement of pedestrians self-organises naturally into lanes with a specific direction. In addition, when a stationary crowd stands in the way and needs to be passed through, pedestrians organise themselves into river-like streams. They act together, more or less automatically, to accomplish a task. How and why they develop these configurations?

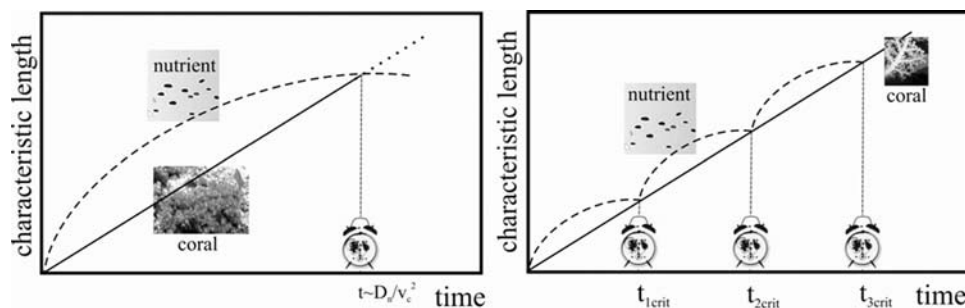


Figure 4: Time evolution of the characteristic length of coral and nutrients (adapted from [10]).

An organised collective behaviour is based on a coordinated and orchestrated interaction between people. They must share information – communicate. The communication can be verbal (i.e. communication that uses words) but also non-verbal (i.e. communication that includes eye contact, body posture and motions, positioning within groups, etc.). Therefore, pedestrians rely on communication to control specific pattern formation. Similar to chemical or electrical signals widely used by bacteria and social insects, pedestrian must rely on their visual positioning (and sometimes in verbal communication) for spatial orientation. Vision guarantees a precise control of the pedestrian position in space and time. Systematic observations of pedestrian revealed that in standard situations (for instance, running to catch a departing bus; panic situation excluded) pedestrian motion reveals regularities. Figure 5 reveals that pedestrians walk ‘fast’ but also ‘slow’. If the interpersonal distance between the pedestrians is large enough (e.g. the density of pedestrians is small enough) the area around the pedestrians is free and they walk with a constant walking speed – domain I – which corresponds to the least-energy consuming walking speed [8]. For a density higher than ~ 1 person/m², the walking speed starts to decrease with the density of pedestrians – domain II [11]. Vision guarantees a precise control of the walking speed, by adjusting the own speed to the speed of neighbouring pedestrians, in order to avoid and reduce physical contact. Why do crowds are able to exhibit dissimilar flow configurations? Why do pedestrians spontaneously self-organise in streams?

Pedestrians flow results of the combination of two mechanisms [10]: streams with a walking speed of 1.34 m/s and standard deviation of 0.26 m/s (i.e. corresponding to the least-energy consuming) and diffusion with a diffusion coefficient of about $1.97\lambda - 0.61$ (with $0.31 \leq \lambda \leq 3.16$ m and $L > 0.31$ m, where λ is the mean interpersonal distance and L the distance to access). Consider pedestrians that proceed from one point to every point of a finite-size territory (e.g. a square, a shopping mall, etc.). As in nature, flows result from the combination of the diffusion and the organised (stream) flow. According to the constructal law, the configuration and function of the flow system must evolve in order to minimise the travel time of the pedestrians. Miguel and Bejan [10] estimate the time of transition from diffusion to streams, t_{crit} , as being $\sim 1.1\lambda - 0.34$. Diffusion is more effective than streams if the diffusion time is lower than t_{crit} . So, diffusion is the preferred flow mechanism for $\lambda > 0.68L + 0.31$ or $L < 1.47\lambda - 0.45$. That is, diffusion is only more competitive for a large mean free interpersonal distance between pedestrians and for accessing small distances (i.e. diffusion has a large spreading resistance). On the other hand, organised (stream) flow is more competitive when mean free interpersonal distance between pedestrians are small and for accessing large distances. These arguments explain the formation of spontaneous lanes (streams) of pedestrians in very crowded open spaces.

In addition, and just as importantly, the argument of minimum resistance also led to the conclusion that streams provide the best flow access when pedestrians try to cross a stationary crowd [10].

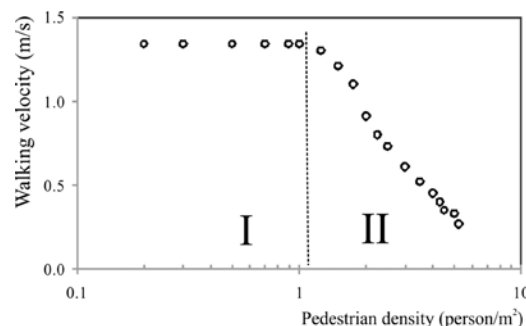


Figure 5: Empirical walking speeds versus density of pedestrians (adapted from [8]).

The ‘rivers of people’ behave similar to river basins (section 2). The stationary crowd is the river basin, and the space vacated by the crossing pedestrians is the eroded river bed. Each pedestrian opens a small space toward the stationary crowd, thereby creating the conditions for a successor to follow. The next pedestrians follow those who are already in motion, giving rise to organised streams that form through the crowd. The streams due to the coalescence of such paths are tree-like structures similar to river branches. The physics meaning of the evolution of the configuration in crowd flow stills the greater flow access (constructal law).

4 CONSTRUCTAL LAW – A UNIFYING PRINCIPLE OF GENERATION OF CONFIGURATION IN EVERYTHING THAT FLOWS AND MOVES

One of the main aims of physics is to interpret different phenomena in a unified view. One of the most influential books in the history of science ‘*Philosophiae Naturalis Principia Mathematica*’ (Mathematical Principles of Natural Philosophy), published in 1687 by Isaac Newton, presented perhaps the first great unifying principle of classical physics – the gravitation law – capable of explaining in one law the motion of the planets, the movement of the tides and the fall of an apple. From these times to now, scientists still seek for unifying principles in nature.

Constructal theory of Adrian Bejan [1, 2], developed first for engineered systems, is the view that the generation of configuration in flow systems is a physics phenomenon. The flow systems possess their configurations ruled under the same principle – the constructal law. In this paper, we present a sequence of results of research available into the constructal generation of flow configuration in both animate and inanimate systems. We presented evidences that shown that animate and inanimate systems acquire configuration in a similar way. To persist in time, the both systems call for maximisation of global performance subject to finite-size constraints. They are not static, they acquire configuration: existing configurations are replaced by configurations that perform better. Therefore, animate and inanimate systems are able to morph. Only systems that have the capacity to evolve acquire configurations that perform better. In the case of animate flow systems (e.g. polyp, bacterium, corals, bacteria, crowds, etc.), to guarantee a precise control of configuration, they need an extra feature – a process of transferring information from one entity to another. They need to rely on communication – between cells and between individuals (polyp, bacterium and pedestrians). The communication can be verbal but also non-verbal (e.g. eye contact, body posture and motions, positioning within groups, but also by chemical signals, electrical impulses, vibration signals, etc.). The last does not elicit specific responses in themselves, but rather operate in a general manner to alter the probability that individuals will respond to other stimuli.

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